Gravitational transition form factors of $N(1535) \to N$

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(Received 13 December 2019; accepted 5 March 2020; published 24 March 2020)

We employ the quark part of the symmetric energy-momentum tensor current to calculate the transition gravitational form factors of the $N(1535) \to N$ by means of the light-cone QCD sum rule formalism. In numerical analysis, we use two different sets of the shape parameters in the distribution amplitudes of the $N(1535)$ baryon and the general form of the nucleon's interpolating current. It is seen that the momentum squared dependence of the gravitational form factors can be well described by the p-pole fit function. The results obtained by using two sets of parameters are found to be quite different from each other, and the $N(1535) \to N$ transition gravitational form factors depend highly on the shape parameters of the distribution amplitudes of the $N(1535)$ state that parametrize the relative orbital angular momentum of the constituent quarks.

DOI: 10.1103/PhysRevD.101.054031

I. INTRODUCTION

The form factors (FFs) are essential parameters for gaining knowledge on the internal organization of the composit particles at low energies. Using various FFs, one can get important information on many quantities, such as size, shape, radius, electrical and magnetic charge distributions, axial and tensor charges, and other mechanical and electromagnetical parameters of hadrons. The gravitational FFs (GFFs) or energy-momentum tensor FFs (EMTFFs) are described by the help of the matrix elements of the symmetric energy-momentum tensor. Just as the Fourier transform of the electromagnetic form factors can be explicated with regard to the spatial distribution of electrical charge and magnetization, the Fourier transform of the gravitational form factors can be explicated with regard to the spatial distribution of momentum, energy, pressure, etc. Hence, the investigation of these FFs attracts significant interest for comprehension of the internal structure of the nucleon. The GFFs of the nucleon have been examined within different theoretical models, such as the chiral quark soliton model [1–11], lattice QCD [12–19], light-cone QCD sum rules (LCSRs) [20,21], the Skyrme model [22,23], chiral perturbation theory [24–29], the bag model [30], and the instant and front forms [31]. Interested readers can find more details about these studies in a recent review [32].

In this study, we extend our previous work on the nucleon's EMTFFs [21] and calculate for the first time (to our knowledge) the transitional gravitational form factors of $N(1535) \to N$ due to the energy-momentum tensor current in the framework of the light-cone QCD sum rule [33–35] using the general form of the interpolating current of nucleon and distribution amplitudes of $N(1535)$. [Hereafter we shall represent the $N(1535)$ particle as N^*.] The main advantage of this method is that it is an analytical method and includes direct QCD parameters. The method involves a two-stage approach. First, the corresponding correlation function is calculated in terms of the quark-gluon properties. Second, it is obtained in terms of hadron properties such as GFFs. When calculating the quark-gluon properties, a connection is established between the low energy processes and the QCD vacuum, which is expressed in terms of distribution amplitudes. In this approach, the hadrons are represented by interpolating currents carrying the same quantum number as the hadrons. These interpolating currents are inserted into the correlation function and the short- (perturbative) and long-distance (nonperturbative) interactions are separated using the operator product expansion (OPE). The hadronic and OPE representations of the same correlation function are then matched. To suppress the unwanted contributions coming from the higher states and continuum, the Borel transformation and continuum subtraction supplied by the quark-hadron duality assumption are

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applied. By choosing some independent Lorentz structures from both sides of the resultant equations, the desired sum rules for the FFs are obtained in terms of hadronic parameters as well as QCD degrees of freedom. The light-cone QCD sum rule approach has been successfully applied to calculate different form factors of hadrons as well as the transition form factors among baryons due to the electromagnetic, axial, and tensor currents at high Q^2 (see, e.g., [36–48]).

The presentation of the manuscript is organized as follows. In Sec. II, we briefly discuss the formalism and calculate the LCSR for the transition gravitational form factors under investigation. In Sec. III, we numerically analyze the $N^* \to N$ transition GFFs. We fix the auxiliary parameters by entering the calculations according to the standard prescriptions of the method. We also use the wave functions and all of the including parameters recently available for the distribution amplitudes (DAs) of the N^* state to find the Q^2 behavior of the form factors in this section. Section IV is dedicated to a discussion of the results achieved for the GFFs. The distribution amplitudes of the N^* state and explicit expressions of the $N^* \to N$ transition GFFs are presented in the appendixes.

II. FORMALISM

The following correlation function is used in the analytical calculations to compute the GFFs of the $N^* \to N$ transition by means of the LCSR:

$$\Pi_{\mu\nu}(p,q) = i\int d^4x e^{iqx}\langle 0|\mathcal{T}[J_N(0)T^q_{\mu\nu}(x)]|N^*(p)\rangle, \qquad (1)$$

where $J_N(0)$ is the interpolating current of the nucleon, the $T^q_{\mu\nu}(x)$ is the quark part of the energy-momentum tensor current, and $\mathcal{T}$ is the time ordering operator.

The main goal in this section is to calculate the abovementioned correlation function in two different languages and apply the prescriptions previously discussed. Thus, the calculations of the hadronic and OPE (QCD) representations of the correlation function are in order.

A. Hadronic representation of the correlation function

We begin to compute the correlator with regards to the hadronic degrees of freedom with the inclusion of the physical features of the hadrons under examination. To that end, we embed intermediate states of the nucleon with the same quantum number of $J_N(0)$ into the correlator. As a result, we get

$$\Pi^{Had}_{\mu\nu}(p,q) = \sum_{s'}\frac{\langle 0|J_N|N(p',s')\rangle\langle N(p',s')|T^q_{\mu\nu}|N^*(p,s)\rangle}{m_N^2 - p'^2} + \cdots, \qquad (2)$$

where the contributions coming from the higher states and continuum are denoted by dots. The expression shown in Eq. (2) can be simplified by introducing the following matrix elements [32]:

$$\langle 0|J_N|N(p',s')\rangle = \lambda_N u_N(p',s'), \qquad (3)$$

$$\langle N(p',s')|T^q_{\mu\nu}|N^*(p,s)\rangle = \bar{u}_N(p',s')\Bigg[A^{N^*-N}(Q^2)\frac{\tilde{P}_\mu\tilde{P}_\nu}{2(m_{N^*}+m_N)} + iJ^{N^*-N}(Q^2)\frac{(\tilde{P}_\mu\sigma_{\nu\rho}+\tilde{P}_\nu\sigma_{\mu\rho})\Delta^\rho}{2(m_{N^*}+m_N)} + D^{N^*-N}(Q^2)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{2(m_{N^*}+m_N)} + \bar{c}^{N^*-N}(Q^2)\frac{m_{N^*}+m_N}{2}g_{\mu\nu}\Bigg]\gamma_5 u_{N^*}(p,s), \qquad (4)$$

where λ_N is residue of the nucleon, $\tilde{P} = p' + p$, $\Delta = p' - p$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu,\gamma_\nu]$, and $Q^2 = -\Delta^2$. Here, $A^{N^*-N}(Q^2)$, $J^{N^*-N}(Q^2)$, $D^{N^*-N}(Q^2)$, and $\bar{c}^{N^*-N}(Q^2)$ are the GFFs. Summation over spin of the nucleon is carried out by

$$\sum_{s'} u_N(p',s')\bar{u}_N(p',s') = \not{p}' + m_N. \qquad (5)$$

Substituting Eqs. (3)–(5) into Eq. (2), we achieve the hadronic representation of the correlation function with respect to the hadronic parameters as

$$\Pi^{Had}_{\mu\nu}(p,q) = \frac{\lambda_N}{m_N^2 - p'^2}(\not{p}' + m_N)\Bigg[A^{N^*-N}(Q^2)\frac{\tilde{P}_\mu\tilde{P}_\nu}{2(m_{N^*}+m_N)} + iJ^{N^*-N}(Q^2)\frac{(\tilde{P}_\mu\sigma_{\nu\rho}+\tilde{P}_\nu\sigma_{\mu\rho})\Delta^\rho}{2(m_{N^*}+m_N)} + D^{N^*-N}(Q^2)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{2(m_{N^*}+m_N)} + \bar{c}^{N^*-N}(Q^2)\frac{m_{N^*}+m_N}{2}g_{\mu\nu}\Bigg]\gamma_5 u_{N^*}(p,s). \qquad (6)$$

From Eq. (6), we can decompose the hadronic side of the correlator in terms of different invariant functions and independent Lorentz structures:

$$\Pi^{Had}_{\mu\nu}(p,q) = \Pi^{Had}_1(Q^2)p'_\mu p'_\nu\gamma_5 + \Pi^{Had}_2(Q^2)p'_\mu p'_\nu\not{q}\gamma_5 + \Pi^{Had}_3(Q^2)g_{\mu\nu}\not{q}\gamma_5 + \Pi^{Had}_4(Q^2)g_{\mu\nu}\gamma_5 + \cdots. \qquad (7)$$

B. QCD representation of the correlation function

To achieve the expression of the correlator in terms of the QCD parameters, explicit forms for the interpolating currents of $J_N(0)$ and $T^q_{\mu\nu}(x)$ are needed. These currents are defined by the following expressions with respect to quark fields:

$$\begin{aligned} J_N(0) &= 2\epsilon^{abc}[[u^{aT}(0)Cd^b(0)]\gamma_5 u^c(0) \\ &\quad + t[u^{aT}(0)C\gamma_5 d^b(0)]u^c(0)], \\ T^q_{\mu\nu}(x) &= \frac{i}{2}[\bar{u}^d(x)\overleftrightarrow{D}_\mu(x)\gamma_\nu u^d(x) \\ &\quad + \bar{d}^e(x)\overleftrightarrow{D}_\mu(x)\gamma_\nu d^e(x) + (\mu \leftrightarrow \nu)], \end{aligned} \tag{8}$$

where the charge conjugation operator, the arbitrary mixing parameter, and the color indices are denoted as C, t, and a, b, c, d, e, respectively. The two-sided covariant derivative is defined as

$$\overleftrightarrow{D}_\mu(x) = \frac{1}{2}[\vec{D}_\mu(x) - \overleftarrow{D}_\mu(x)], \tag{9}$$

with

$$\vec{D}_\mu(x) = \vec{\partial}_\mu(x) + igA_\mu(x), \tag{10}$$

$$\overleftarrow{D}_\mu(x) = \overleftarrow{\partial}_\mu(x) - igA_\mu(x), \tag{11}$$

where A_μ is the gluon field. We neglect the gluon field contributions, i.e., the gluonic part of the EMT, since considering these contributions requires information about quark-gluon mixed DAs of the N^* state, which unfortunately are not available. Therefore, in this work, we shall take note of the quark part of the energy-momentum tensor current in Eq. (8). We insert the interpolating currents $J_N(0)$ and $T^q_{\mu\nu}(x)$ into the correlator and carry out the necessary contractions with the help of the Wick theorem. Consequently, we obtain

$$\begin{aligned} \Pi^{\rm QCD}_{\mu\nu}(p,q) = -\int d^4x e^{iqx}[\{&(\gamma_5)_{\gamma\delta}C_{\alpha\beta}(\overleftrightarrow{D}_\mu(x)\gamma_\nu)_{\omega\rho} + t(I)_{\gamma\delta}(C\gamma_5)_{\alpha\beta}(\overleftrightarrow{D}_\mu(x)\gamma_\nu)_{\omega\rho} + (\gamma_5)_{\gamma\delta}C_{\alpha\beta}(\overleftrightarrow{D}_\nu(x)\gamma_\mu)_{\omega\rho} \\ &+ t(I)_{\gamma\delta}(C\gamma_5)_{\alpha\beta}(\overleftrightarrow{D}_\nu(x)\gamma_\mu)_{\omega\rho}\}\{(\delta^\alpha_\sigma\delta^\rho_\theta\delta^\beta_\phi S_u(-x)_{\delta\omega} + \delta^\delta_\sigma\delta^\rho_\theta\delta^\beta_\phi S_u(-x)_{\alpha\omega})\langle 0|\epsilon^{abc}u^a_\sigma(0)u^b_\theta(x)d^c_\phi(0)|N^*(p)\rangle \\ &+ \delta^\alpha_\sigma\delta^\delta_\theta\delta^\rho_\phi S_d(-x)_{\beta\omega}\langle 0|\epsilon^{abc}u^a_\sigma(0)u^b_\theta(0)d^c_\phi(x)|N^*(p)\rangle\}], \end{aligned} \tag{12}$$

where $S_q(x)$ is propagator of the light $q = u$, d quarks and is identified as

$$S_q(x) = \frac{1}{2\pi^2x^2}\left(i\frac{\not{x}}{x^2} - \frac{m_q}{2}\right) - \frac{\langle\bar{q}q\rangle}{12}\left(1 - i\frac{m_q\not{x}}{4}\right) - \frac{\langle\bar{q}\sigma.Gq\rangle}{192}x^2\left(1 - i\frac{m_q\not{x}}{6}\right) - \frac{ig_s}{32\pi^2x^2}G^{\mu\nu}(x)[\not{x}\sigma_{\mu\nu} + \sigma_{\mu\nu}\not{x}]. \tag{13}$$

It should be noted here that we are working with an $m_q = 0$ limit; therefore, the terms proportional to the quark mass do not give any contribution. The terms proportional with quark ($\langle\bar{q}q\rangle$) and mixed ($\langle\bar{q}\sigma.Gq\rangle$) condensates are killed by performing a Borel transformation. The contributions coming from the terms corresponding to the gluon strength field tensor ($G_{\mu\nu}$) are expected to be small [49], which are relevant to the distribution amplitudes of four- and five-particles. Therefore, we can neglect the contributions of these terms in calculations as well. As a result, only the first term of the light quark propagator contributes to our computations. The $\langle 0|\epsilon^{abc}u^a_\sigma(a_1x)u^b_\theta(a_2x)d^c_\phi(a_3x)|N^*(p)\rangle$ matrix element is the expression containing the distribution amplitudes of the N^* state, and it is required for further calculations. The explicit form of this matrix element in terms of the related DAs together with the explicit forms of DAs for N^* are given in Appendix A. Using the distribution amplitudes of the N^* state and applying the integration over x, the QCD representation of the correlation function is acquired as

$$\begin{aligned} \Pi^{\rm QCD}_{\mu\nu}(p,q) &= \Pi^{\rm QCD}_1(Q^2)p'_\mu p'_\nu\gamma_5 + \Pi^{\rm QCD}_2(Q^2)p'_\mu p'_\nu\not{q}\gamma_5 \\ &\quad + \Pi^{\rm QCD}_3(Q^2)g_{\mu\nu}\not{q}\gamma_5 + \Pi^{\rm QCD}_4(Q^2)g_{\mu\nu}\gamma_5 \\ &\quad + \cdots. \end{aligned} \tag{14}$$

The explicit expressions of $\Pi^{\rm QCD}_1(Q^2)$, $\Pi^{\rm QCD}_2(Q^2)$, $\Pi^{\rm QCD}_3(Q^2)$, and $\Pi^{\rm QCD}_4(Q^2)$ are presented in Appendix B.

C. Light-cone QCD sum rules for the $N^* \to N$ transition

The required light-cone QCD sum rules for the $N^* \to N$ transition GFFs are obtained by matching the coefficients of different Lorentz structures from both the hadronic and QCD representations of the correlator.

We shall note that we employ the structures $p'_\mu p'_\nu\gamma_5$, $p'_\mu p'_\nu\not{q}\gamma_5$, $q_\mu q_\nu\not{q}\gamma_5$, and $g_{\mu\nu}\gamma_5$ to find the light-cone QCD sum rules for the transition GFFs, $A^{N^*-N}(Q^2)$, $J^{N^*-N}(Q^2)$, $D^{N^*-N}(Q^2)$, and $\bar{c}^{N^*-N}(Q^2)$, respectively. Hence,

TABLE I. Numerical values of the shape parameters of the distribution amplitudes for the N^* state at renormalization scale $\mu^2 = 2.0\ \mathrm{GeV}^2$. Besides these values, we use $\lambda_1^N m_N = -3.88(2)(19)\times 10^{-2}\ \mathrm{GeV}^3$ and $\lambda_2^{N^*} m_{N^*} = 8.97(45)\times 10^{-2}\ \mathrm{GeV}^3$, given in Ref. [52] at renormalization $\mu^2 = 4.0\ \mathrm{GeV}^2$, by rescaling to $\mu^2 = 2.0\ \mathrm{GeV}^2$.

Model	$\lvert\lambda_1^{N^*}/\lambda_1^N\rvert$	$f_{N^*}/\lambda_1^{N^*}$	φ_{10}	φ_{11}	φ_{20}	φ_{21}	φ_{22}	η_{10}	η_{11}
LCSR 1	0.633	0.027	0.36	−0.95	0	0	0	0	0.94
LCSR 2	0.633	0.027	0.37	−0.96	0	0	0	−0.29	0.23

$$\frac{\lambda_N}{m_N^2 - p'^2} A^{N^*-N}(Q^2) = -\frac{m_N + m_{N^*}}{2m_{N^*}} \Pi_1^{\mathrm{QCD}}(Q^2), \tag{15}$$

$$\frac{\lambda_N}{m_N^2 - p'^2} J^{N^*-N}(Q^2) = \frac{m_N + m_{N^*}}{2} \Pi_2^{\mathrm{QCD}}(Q^2), \tag{16}$$

$$\frac{\lambda_N}{m_N^2 - p'^2} D^{N^*-N}(Q^2) = -2(m_N + m_{N^*}) \Pi_3^{\mathrm{QCD}}(Q^2), \tag{17}$$

$$\frac{\lambda_N}{m_N^2 - p'^2} \bar{c}^{N^*-N}(Q^2) = -\frac{2}{m_{N^*}^2} \Pi_4^{\mathrm{QCD}}(Q^2). \tag{18}$$

For the calculation of the $N^* \to N$ transition GFFs, the residue of the nucleon λ_N is needed as well. The residue of the nucleon is determined from two point sum rules [42]:

$$\lambda_N^2 e^{-\frac{m_N^2}{M^2}} = \frac{M^6}{256\pi^4}(5+2t+t^2)E_2(z) - \frac{\langle\bar{q}q\rangle^2}{6} \times \left\{6(1-t^2)-(1-t)^2-\frac{m_0^2}{4M^2}[12(1-t^2)-(1-t)^2]\right\}, \tag{19}$$

where

$$z = s_0/M^2$$

and

$$E_n(z) = 1 - e^{-z}\sum_{i=0}^{n}\frac{z^i}{i!}.$$

III. NUMERICAL RESULTS

This section is dedicated to the numerical analysis of the $N^* \to N$ transition GFFs. To this end, we need distribution amplitudes of the N^* state. The explicit expressions of these distribution amplitudes are given in Appendix A. For further calculations, we need the shape parameters of the distribution amplitudes of the N^* state, which are presented in Table I. Additionally, we use $m_N = 0.94$ GeV, $m_{N^*} = 1.51 \pm 0.01$ GeV [50], $m_q = 0$, $\langle\bar{q}q\rangle = (-0.24\pm 0.01)^3\ \mathrm{GeV}^3$, and $m_0^2 = 0.8 \pm 0.1\ \mathrm{GeV}^2$ [51].

The LCSRs for the $N^* \to N$ transition GFFs also include some auxiliary parameters: the mixing parameter t, the continuum threshold s_0, and the Borel mass parameter square M^2. The GFFs should not be affected by the changes of these parameters much. Hence, we search for working windows for these auxiliary parameters such that, in these working windows, the GFFs depend relatively weakly on these parameters. The mixing parameter t is chosen such that the estimation of the GFFs is independent of the value of t in its working region. From the numerical calculations, it is obtained that in the region $-0.2 \le \cos\theta \le -0.45$ the GFFs weakly depend on t, where $\tan\theta = t$. The working region for continuum threshold s_0 is acquired while taking into account the fact that the GFFs are almost insensitive with respect to its changes as well. We choose the s_0 in the interval $2.5\ \mathrm{GeV}^2 \le s_0 \le 3.0\ \mathrm{GeV}^2$. We use the following steps to achieve the working interval for the Borel mass parameter M^2. The lower cutoff of M^2 is obtained while demanding that the perturbative part exceeds the nonperturbative one and that the series of nonperturbative terms are convergent. The upper cutoff of M^2 is acquired using the condition that the contributions of higher states and continuum should be less than the ground state contribution. These requirements are both fulfilled when M^2 varies in the interval $2.0\ \mathrm{GeV}^2 \le M^2 \le 3.5\ \mathrm{GeV}^2$.

In Figs. 1–3, we plot the dependences of the GFFs $A^{N^*-N}(Q^2)$, $J^{N^*-N}(Q^2)$, $D^{N^*-N}(Q^2)$, and $\bar{c}^{N^*-N}(Q^2)$ on different parameters for the various values of the arbitrary mixing parameter t and other parameters in their working regions for two sets of the distribution amplitudes for the N^* baryon. As can be seen from Figs. 1 and 2, the obtained values of the GFFs are approximately independent of the continuum threshold s_0 and Borel mass parameter M^2 on their working regions. Therefore, for numerical calculations of the central values of GFFs, we shall use the central values of the M^2 and s_0. The LCSR method is reliable only when $Q^2 > 1.0\ \mathrm{GeV}^2$. However, the mass corrections of the distribution amplitudes $\sim m_{N^*}^2/Q^2$ become quite large for $Q^2 < 2.0\ \mathrm{GeV}^2$—namely, the LCSRs turn out to be unreliable. Therefore, for GFFs, we expect the LCSRs to operate efficiently and effectively in the $2.0\ \mathrm{GeV}^2 \le Q^2 \le 6.0\ \mathrm{GeV}^2$ region. In Fig. 3, we present the Q^2 dependency of the GFFs on the fixed Borel mass parameter and continuum threshold and various values of the arbitrary mixing parameter t. We see that GFFs vary smoothly in terms of Q^2, as expected. We also observe that the GFFs $A^{N^*-N}(Q^2)$, $J^{N^*-N}(Q^2)$, and $D^{N^*-N}(Q^2)$ are sensitive to the variations

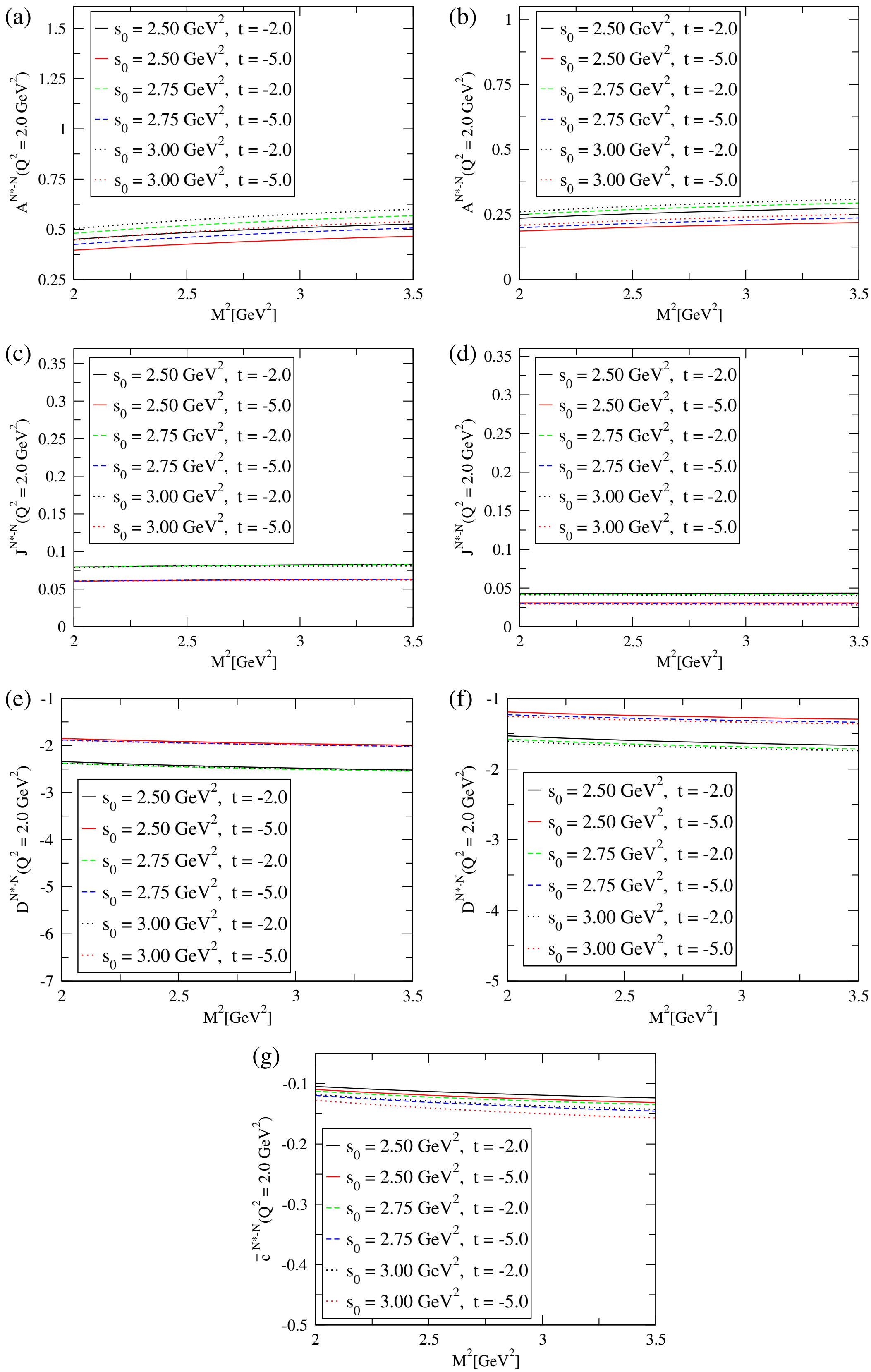

FIG. 1. The dependence of the GFFs of the $N^* \to N$ transition on the Borel mass parameter M^2 at $Q^2 = 2.0\ \mathrm{GeV}^2$ and various values of continuum threshold s_0 and arbitrary mixing parameter t at their working region: (a), (c), (e), and (g) for LCSR 1, and (b), (d), and (f) for LCSR 2.

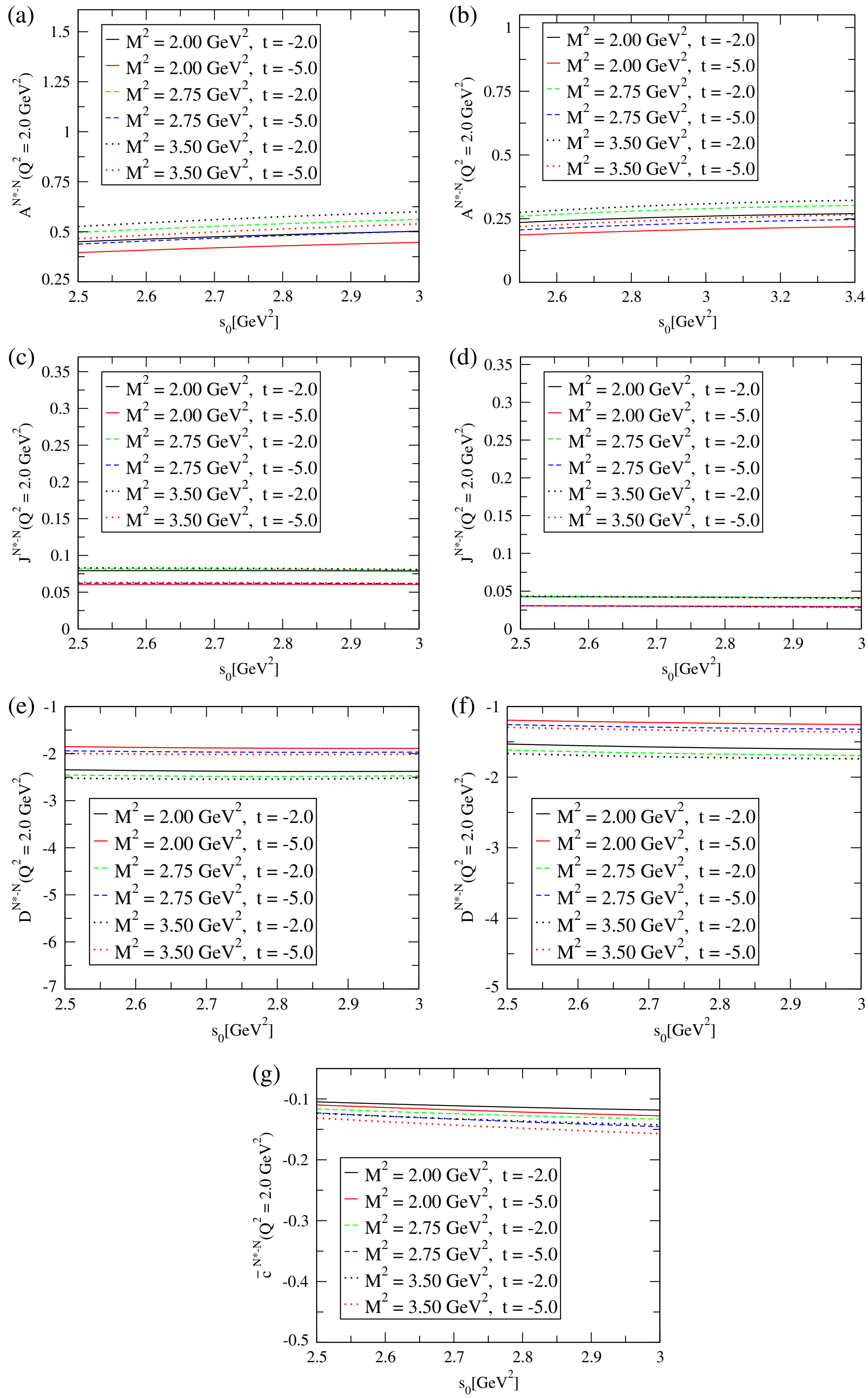

FIG. 2. The dependence of the GFFs of the $N^* \to N$ transition on the continuum threshold s_0 at $Q^2 = 2.0\ \mathrm{GeV}^2$, and various values of the Borel mass parameter M^2 and the arbitrary mixing parameter t at their working region: (a), (c), (e), and (g) for LCSR 1, and (b), (d), and (f) for LCSR 2.

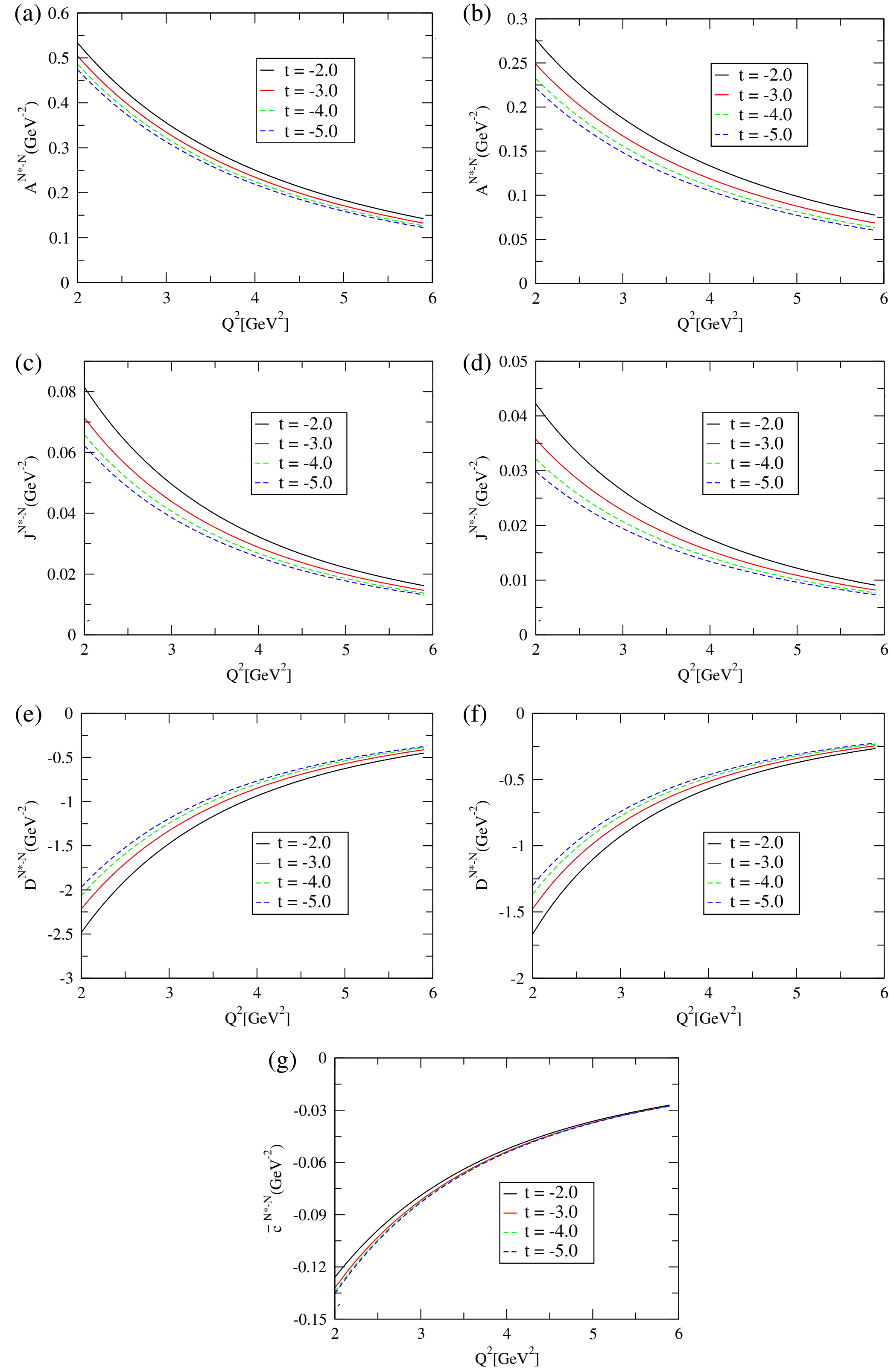

FIG. 3. The dependence of the GFFs of the $N^* \to N$ transition on Q^2 at $M^2 = 2.75\ \mathrm{GeV}^2$, $s_0 = 2.75\ \mathrm{GeV}^2$, and various values of the arbitrary mixing parameter t: (a), (c), (e), and (g) for LCSR 1, and (b), (d), and (f) for LCSR 2.

TABLE II. The numerical results for the parameters of the GFFs obtained by using the p-pole fit functions.

	LCSR 1			LCSR 2		
Form factors	$\mathcal{F}(0)$	m_p (GeV)	p	$\mathcal{F}(0)$	m_p (GeV)	p
$A^{N^*-N}(Q^2)$	1.33 ± 0.13	1.30 ± 0.10	3.0–3.4	0.63 ± 0.10	1.32 ± 0.10	3.0–3.4
$J^{N^*-N}(Q^2)$	0.27 ± 0.07	1.13 ± 0.10	3.0–3.4	0.13 ± 0.05	1.17 ± 0.11	2.9–3.3
$D^{N^*-N}(Q^2)$	-8.20 ± 2.02	1.14 ± 0.10	3.6–4.0	-6.91 ± 1.80	1.02 ± 0.09	3.2–3.6
$\bar{c}^{N^*-N}(Q^2)$	-0.40 ± 0.06	1.18 ± 0.10	3.0–3.5	...	...	...

of the mixing parameter t for both sets. The $\bar{c}^{N^*-N}(Q^2)$ form factor is obtained using only the first set of the distribution amplitudes. The second set of the distribution amplitudes gives unphysical results for this form factor (the form factor changes its sign in the region under consideration); therefore, it is not presented.

To extend the behavior of GFFs to the region $0 \leq Q^2 < 2$, we need to use some fit parametrizations. Our numerical calculations show that the GFFs of the $N^* \to N$ transition can be properly characterized using the p-pole fit function

$$\mathcal{F}(Q^2) = \frac{\mathcal{F}(0)}{(1 + Q^2/(pm_p^2))^p}. \tag{20}$$

Our results for the fit parameters of the $N^* \to N$ transition GFFs are presented in Table II. The results, obtained by using two sets, are found to be quite different from each other. As can be seen in Table I, the main difference between the two sets of distribution amplitudes is the numerical values for the shape parameters η_{10} and η_{11}, which are connected to the three quark wave functions of the p-wave N^* baryon. As a result, we find that the $N^* \to N$ transition GFFs depend strongly on the input parameters of the distribution amplitudes of the N^* baryon that parametrize the relative orbital angular momentum of the quarks. There are no theoretical predictions for the transition GFFs under study in the literature to be compared to our results. The $D(Q^2)$ GFF of the N^* has recently been obtained by means of the bag model, $D(Q^2 = 0) = -12.97$ [30]. Any future experimental data will help us gain useful knowledge about the DAs of the N^* state and their nature and internal structure.

IV. SUMMARY AND CONCLUDING REMARKS

We applied the quark part of the symmetric energy-momentum tensor current to compute, for the first time, the GFFs of the $N(1535) \to N$ transition with the help of the LCSR approach. Studying the GFFs of the particles result in valuable knowledge about the total angular momentum, the spatial distribution of energy, pressure, and shear forces inside the particles, etc. In our numerical analysis, we used two different sets of shape parameters in the distribution amplitudes of the $N(1535)$ state and took into account the most general form of the nucleon's interpolating current. It was seen that the momentum squared dependence of the gravitational form factors can be well described by a p-pole fit function. The results obtained by using two sets were found to be quite different from each other. We found that the values of the $N(1535) \to N$ transition GFFs depend greatly on the input parameters of the distribution amplitudes of the $N(1535)$ state that parametrize the relative orbital angular momentum of the quarks. As previously mentioned, we calculated the $N(1535) \to N$ transition GFFs for the first time in the literature. Therefore, there are no experimental data or theoretical predictions to be compared with the results obtained in this study. Calculations of the GFFs from different methods and approaches are of great importance. Such calculations and a comparison of the obtained results with each other will not only help us get useful information on the DAs of the N^* state but also experimental groups for measuring the values of the related GFFs.

ACKNOWLEDGMENTS

U. Ö. acknowledges the support of the Scientific and Technological Research Council of Turkey (TUBITAK) provided through the 2218-National Postdoctoral Research Scholarship Programme.

APPENDIX A: DISTRIBUTION AMPLITUDES OF N^* THE STATE

In this appendix, we present the explicit forms of N^* DAs [53]:

$$\begin{aligned}
&4\langle 0|\epsilon^{abc} u_\alpha^a(a_1 x) d_\beta^b(a_2 x) d_\gamma^c(a_3 x)|N^*(p)\rangle \\
&= \mathcal{S}_1 m_{N^*} C_{\alpha\beta} N_\gamma^* - \mathcal{S}_2 m_{N^*}^2 C_{\alpha\beta}(\not{x} N^*)_\gamma + \mathcal{P}_1 m_{N^*}(\gamma_5 C)_{\alpha\beta}(\gamma_5 N^*)_\gamma + \mathcal{P}_2 m_{N^*}^2 (\gamma_5 C)_{\alpha\beta}(\gamma_5 \not{x} N^*)_\gamma \\
&\quad - \left(\mathcal{V}_1 + \frac{x^2 m_{N^*}^2}{4}\mathcal{V}_1^M\right)(\not{p} C)_{\alpha\beta} N_\gamma^* + \mathcal{V}_2 m_{N^*}(\not{p} C)_{\alpha\beta}(\not{x} N^*)_\gamma + \mathcal{V}_3 m_{N^*}(\gamma_\mu C)_{\alpha\beta}(\gamma^\mu N^*)_\gamma - \mathcal{V}_4 m_{N^*}^2 (\not{x} C)_{\alpha\beta} N_\gamma^*
\end{aligned}$$

$$
\begin{aligned}
&- \mathcal{V}_5 m_{N^*}^2 (\gamma_\mu C)_{\alpha\beta}(i\sigma^{\mu\nu}x_\nu N^*)_\gamma + \mathcal{V}_6 m_{N^*}^3 (\not{x}C)_{\alpha\beta}(\not{x}N^*)_\gamma - \left(\mathcal{A}_1 + \frac{x^2 m_{N^*}^2}{4}\mathcal{A}_1^M\right)(\not{p}\gamma_5 C)_{\alpha\beta}(\gamma N^*)_\gamma \\
&+ \mathcal{A}_2 m_{N^*}(\not{p}\gamma_5 C)_{\alpha\beta}(\not{x}\gamma_5 N^*)_\gamma + \mathcal{A}_3 m_{N^*}(\gamma_\mu\gamma_5 C)_{\alpha\beta}(\gamma^\mu\gamma_5 N^*)_\gamma \\
&- \mathcal{A}_4 m_{N^*}^2 (\not{x}\gamma_5 C)_{\alpha\beta}(\gamma_5 N^*)_\gamma - \mathcal{A}_5 m_{N^*}^2 (\gamma_\mu\gamma_5 C)_{\alpha\beta}(i\sigma^{\mu\nu}x_\nu\gamma_5 N^*)_\gamma + \mathcal{A}_6 m_{N^*}^3 (\not{x}\gamma_5 C)_{\alpha\beta}(\not{x}\gamma_5 N^*)_\gamma \\
&- \left(\mathcal{T}_1 + \frac{x^2 m_{N^*}^2}{4}\mathcal{T}_1^M\right)(i\sigma_{\mu\nu}p_\nu C)_{\alpha\beta}(\gamma^\mu N^*)_\gamma + \mathcal{T}_2 m_{N^*}(i\sigma_{\mu\nu}x^\mu p^\nu C)_{\alpha\beta}N^*_\gamma + \mathcal{T}_3 m_{N^*}(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}N^*)_\gamma \\
&+ \mathcal{T}_4 m_{N^*}(\sigma_{\mu\nu}p^\nu C)_{\alpha\beta}(\sigma^{\mu\rho}x_\rho N^*)_\gamma - \mathcal{T}_5 m_{N^*}^2 (i\sigma_{\mu\nu}x^\nu C)_{\alpha\beta}(\gamma^\mu N^*)_\gamma - \mathcal{T}_6 m_{N^*}^2 (i\sigma_{\mu\nu}x^\mu p^\nu C)_{\alpha\beta}(\not{x}N^*)_\gamma \\
&- \mathcal{T}_7 m_{N^*}^2 (\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}\not{x}N^*)_\gamma + \mathcal{T}_8 m_{N^*}^3 (\sigma_{\mu\nu}x^\nu C)_{\alpha\beta}(\sigma^{\mu\rho}x_\rho N^*)_\gamma .
\end{aligned}
$$

The calligraphic functions can be represented with respect to the functions of the specific twists as

$$
\begin{aligned}
\mathcal{S}_1 &= S_1, \qquad 2p.x\mathcal{S}_2 = S_1 - S_2, \\
\mathcal{P}_1 &= P_1, \qquad 2p.x\mathcal{P}_2 = P_1 - P_2, \\
\mathcal{V}_1 &= V_1, \qquad 2p.x\mathcal{V}_2 = V_1 - V_2 - V_3, \\
2\mathcal{V}_3 &= V_3, \qquad 4p.x\mathcal{V}_4 = -2V_1 + V_3 + V_4 + 2V_5, \\
4p.x\mathcal{V}_5 &= V_4 - V_3, \qquad 4(p.x)^2\mathcal{V}_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6 \\
\mathcal{A}_1 &= A_1, \qquad 2p.x\mathcal{A}_2 = -A_1 + A_2 - A_3, \\
2\mathcal{A}_3 &= A_3, \qquad 4p.x\mathcal{A}_4 = -2A_1 - A_3 - A_4 + 2A_5, \\
4p.x\mathcal{A}_5 &= A_3 - A_4, \qquad 4(p.x)^2\mathcal{A}_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6 \\
\mathcal{T}_1 &= T_1, \qquad 2p.x\mathcal{T}_2 = T_1 + T_2 - 2T_3, \\
2\mathcal{T}_3 &= T_7, \qquad 2p.x\mathcal{T}_4 = T_1 - T_2 - 2T_7, \\
2p.x\mathcal{T}_5 &= -T_1 + T_5 + 2T_8, \qquad 4(p.x)^2\mathcal{T}_6 = 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8, \\
4p.x\mathcal{T}_7 &= T_7 - T_8, \qquad 4(p.x)^2\mathcal{T}_8 = -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8,
\end{aligned}
$$

where V_i, A_i, T_i, S_i, and P_i are vector, axial-vector, tensor, scalar, and pseudoscalar distribution amplitudes, respectively. The explicit expressions of these functions are given as follows:

$$
\begin{aligned}
&V_1(x_i,\mu) = 120x_1x_2x_3[\phi_3^0(\mu) + \phi_3^+(\mu)(1-3x_3)], \\
&V_2(x_i,\mu) = 24x_1x_2[\phi_4^0(\mu) + \phi_4^+(\mu)(1-5x_3)], \\
&V_3(x_i,\mu) = 12x_3\{\psi_4^0(\mu)(1-x_3) + \psi_4^-(\mu)[x_1^2 + x_2^2 - x_3(1-x_3)] + \psi_4^+(\mu)(1-x_3-10x_1x_2)\}, \\
&V_4(x_i,\mu) = 3\{\psi_5^0(\mu)(1-x_3) + \psi_5^-(\mu)[2x_1x_2 - x_3(1-x_3)] + \psi_5^+(\mu)[1-x_3-2(x_1^2+x_2^2)]\}, \\
&V_5(x_i,\mu) = 6x_3[\phi_5^0(\mu) + \phi_5^+(\mu)(1-2x_3)], \\
&V_6(x_i,\mu) = 2[\phi_6^0(\mu) + \phi_6^+(\mu)(1-3x_3)], \\
&A_1(x_i,\mu) = 120x_1x_2x_3\phi_3^-(\mu)(x_2-x_1), \\
&A_2(x_i,\mu) = 24x_1x_2\phi_4^-(\mu)(x_2-x_1), \\
&A_3(x_i,\mu) = 12x_3(x_2-x_1)\{(\psi_4^0(\mu) + \psi_4^+(\mu)) + \psi_4^-(\mu)(1-2x_3)\}, \\
&A_4(x_i,\mu) = 3(x_2-x_1)\{-\psi_5^0(\mu) + \psi_5^+(\mu)(1-2x_3) + \psi_5^-(\mu)x_3\}, \\
&A_5(x_i,\mu) = 6x_3(x_2-x_1)\phi_5^-(\mu) \\
&A_6(x_i,\mu) = 2(x_2-x_1)\phi_6^-(\mu), \\
&T_1(x_i,\mu) = 120x_1x_2x_3\left[\phi_3^0(\mu) + \frac{1}{2}(\phi_3^- - \phi_3^+)(\mu)(1-3x_3)\right],
\end{aligned}
$$

$$T_2(x_i,\mu)=24x_1x_2[\xi_4^0(\mu)+\xi_4^+(\mu)(1-5x_3)],$$
$$T_3(x_i,\mu)=6x_3\{(\xi_4^0+\phi_4^0+\psi_4^0)(\mu)(1-x_3)+(\xi_4^-+\phi_4^--\psi_4^-)(\mu)[x_1^2+x_2^2-x_3(1-x_3)]+(\xi_4^++\phi_4^++\psi_4^+)(\mu)(1-x_3-10x_1x_2)\},$$
$$T_4(x_i,\mu)=\frac{3}{2}\{(\xi_5^0+\phi_5^0+\psi_5^0)(\mu)(1-x_3)+(\xi_5^-+\phi_5^--\psi_5^-)(\mu)[2x_1x_2-x_3(1-x_3)]+(\xi_5^++\phi_5^++\psi_5^+)(\mu)(1-x_3-2(x_1^2+x_2^2))\},$$
$$T_5(x_i,\mu)=6x_3[\xi_5^0(\mu)+\xi_5^+(\mu)(1-2x_3)],$$
$$T_6(x_i,\mu)=2\left[\phi_6^0(\mu)+\frac{1}{2}(\phi_6^--\phi_6^+)(\mu)(1-3x_3)\right],$$
$$T_7(x_i,\mu)=6x_3\{(-\xi_4^0+\phi_4^0+\psi_4^0)(\mu)(1-x_3)+(-\xi_4^-+\phi_4^--\psi_4^-)(\mu)[x_1^2+x_2^2-x_3(1-x_3)](1-x_3)]$$
$$+(-\xi_4^++\phi_4^++\psi_4^+)(\mu)(1-x_3-10x_1x_2)\},$$
$$T_8(x_i,\mu)=\frac{3}{2}\{(-\xi_5^0+\phi_5^0+\psi_5^0)(\mu)(1-x_3)+(-\xi_5^-+\phi_5^--\psi_5^-)(\mu)[2x_1x_2-+x_3(1-x_3)](1-x_3)]$$
$$+(-\xi_5^++\phi_5^++\psi_5^+)(\mu)(1-x_3-2(x_1^2+x_2^2))\},$$
$$S_1(x_i,\mu)=6x_3(x_2-x_1)[(\xi_4^0+\phi_4^0+\psi_4^0+\xi_4^++\phi_4^++\psi_4^+)(\mu)+(\xi_4^-+\phi_4^--\psi_4^-)(\mu)(1-2x_3)],$$
$$S_2(x_i,\mu)=\frac{3}{2}(x_2-x_1)[-(\psi_5^0+\phi_5^0+\xi_5^0)(\mu)+(\xi_5^-+\phi_5^--\psi_5^-)(\mu)x_3+(\xi_5^++\phi_5^++\psi_5^0)(\mu)(1-2x_3)],$$
$$P_1(x_i,\mu)=6x_3(x_2-x_1)[(\xi_4^0-\phi_4^0-\psi_4^0+\xi_4^+-\phi_4^+-\psi_4^+)(\mu)+(\xi_4^--\phi_4^-+\psi_4^-)(\mu)(1-2x_3)],$$
$$P_2(x_i,\mu)=\frac{3}{2}(x_2-x_1)[(\psi_5^0+\psi_5^0-\xi_5^0)(\mu)+(\xi_5^--\phi_5^-+\psi_5^-)(\mu)x_3+(\xi_5^+-\phi_5^+-\psi_5^+)(\mu)(1-2x_3)],$$
$$\mathcal{V}_1^M(x_2)=\int_0^{1-x_2}dx_1V_1^M(x_1,x_2,1-x_1-x_2)=\frac{x_2^2}{24}[f_{N^*}C_f^u(x_2)+\lambda_1^{N^*}C_\lambda^u(x_2)],$$
$$\mathcal{A}_1^M(x_2)=\int_0^{1-x_2}dx_1A_1^M(x_1,x_2,1-x_1-x_2)=\frac{x_2^2}{24}(1-x_2)^3[f_{N^*}D_f^u(x_2)+\lambda_1^{N^*}D_\lambda^u(x_2)],$$
$$\mathcal{T}_1^M(x_2)=\int_0^{1-x_2}dx_1T_1^M(x_1,x_2,1-x_1-x_2)=\frac{x^2}{48}[f_{N^*}E_f^u(x_2)+\lambda_1^{N^*}E_\lambda^u(x_2)].$$

The subsequent functions come across to the above DAs, and they can be parametrized with respect to the independent parameters, such as f_{N^*}, λ_1, λ_2, f_1^u, f_1^d, f_2^d, A_1^u, and V_1^d:

$$\phi_3^0=\phi_6^0=f_{N^*},$$
$$\phi_4^0=\phi_5^0=\frac{1}{2}(f_{N^*}+\lambda_1^{N^*}),$$
$$\xi_4^0=\xi_5^0=\frac{1}{6}\lambda_2^{N^*},$$
$$\psi_4^0=\psi_5^0=\frac{1}{2}(f_{N^*}-\lambda_1^{N^*}),$$
$$\phi_3^-=\frac{21}{2}f_{N^*}A_1^u,\qquad \phi_3^+=\frac{7}{2}f_{N^*}(1-V_1^d),$$
$$\phi_4^+=\frac{1}{4}[f_{N^*}(3-10V_1^d)+\lambda_1^{N^*}(3-10f_1^d)],$$
$$\phi_4^-=-\frac{5}{4}[f_{N^*}(1-2A_1^u)-\lambda_1^{N^*}(1-2f_1^d-4f_1^u)],$$
$$\psi_4^+=-\frac{1}{4}[f_{N^*}(2+5A_1^u-5V_1^d)-\lambda_1^{N^*}(2-5f_1^d-5f_1^u)],$$
$$\psi_4^-=\frac{5}{4}[f_{N^*}(2-A_1^u-3V_1^d)-\lambda_1^{N^*}(2-7f_1^d+f_1^u)],$$

$$\xi_4^+ = \frac{1}{16}\lambda_2^{N^*}(4-15f_2^d), \quad \xi_4^- = \frac{5}{16}\lambda_2^{N^*}(4-15f_2^d),$$
$$\phi_5^+ = -\frac{5}{6}[f_{N^*}(3+4V_1^d)-\lambda_1^{N^*}(1-4f_1^d)], \quad \phi_5^- = -\frac{5}{3}[f_{N^*}(1-2A_1^u)-\lambda_1^{N^*}(f_1^d-f_1^u)],$$
$$\psi_5^+ = -\frac{5}{6}[f_{N^*}(5+2A_1^u-2V_1^d)-\lambda_1^{N^*}(1-2f_1^d-2f_1^u)],$$
$$\psi_5^- = \frac{5}{3}[f_{N^*}(2-A_1^u-3V_1^d)+\lambda_1^{N^*}(f_1^d-f_1^u)],$$
$$\xi_5^+ = \frac{5}{36}\lambda_2^{N^*}(2-9f_2^d), \quad \xi_5^- = -\frac{5}{4}\lambda_2^{N^*}f_2^d,$$
$$\phi_6^+ = \frac{1}{2}[f_{N^*}(1-4V_1^d)-\lambda_1^{N^*}(1-2f_1^d)],$$
$$\phi_6^- = \frac{1}{2}[f_{N^*}(1+4A_1^d)+\lambda_1^{N^*}(1-4f_1^d-2f_1^u)],$$
$$C_f^u(x_2) = (1-x_2)^3[113+495x_2-552x_2^2-10A_1^u(1-3x_2)+2V_1^d(113-951x_2+828x_2^2)],$$
$$C_\lambda^u(x_2) = -(1-x_2)^3[13-20f_1^d+3x_2+10f_1^u(1-3x_2)],$$
$$D_f^u(x_2) = 11+45x_2-2A_1^u(113-951x_2+828x_2^2)+10V_1^d(1-30x_2),$$
$$D_\lambda^u(x_2) = 29-45x_2-10f_1^u(7-9x_2)-20f_1^d(5-6x_2),$$
$$E_f^u(x_2) = -\{(1-x_2)[3(439+71x_2-621x_2^2+587x_2^3-184x_2^4)+4A_1^u(1-x_2)^2(59-483x_2+414x_2^2)$$
$$-4V_1^d(1301-619x_2-769x_2^2+1161x_2^3-414x_2^4)]\}-12(73-220V_1^d)\ln[x_2],$$
$$E_\lambda^u(x_2) = -\{(1-x_2)[5-211x_2+281x_2^2-111x_2^3+10(1+61x_2-83x_2^2+33x_2^3)f_1^d-40(1-x_2)^2(2-3x_2)f_1^u]\}$$
$$-12(3-10f_1^d)\ln[x_2],$$

where the parameters $A_1^u, V_1^d, f_1^d, f_1^u$, and f_2^d are defined as [53]

$$A_1^u = \varphi_{10}+\varphi_{11}, \qquad V_1^d = \frac{1}{3}-\varphi_{10}+\frac{1}{3}\varphi_{11}, \qquad f_1^u = \frac{1}{10}-\frac{1}{6}\frac{f_{N^*}}{\lambda_1^{N^*}}-\frac{3}{5}\eta_{10}-\frac{1}{3}\eta_{11},$$
$$f_1^d = \frac{3}{10}-\frac{1}{6}\frac{f_{N^*}}{\lambda_1^{N^*}}+\frac{1}{5}\eta_{10}-\frac{1}{3}\eta_{11}, \qquad f_2^d = \frac{4}{15}+\frac{2}{5}\xi_{10}.$$

APPENDIX B: EXPLICIT FORMS OF THE FUNCTIONS Π_i FOR THE $N^* \to N$ TRANSITION

In this appendix, we present the explicit forms of the functions derived in the QCD side:

$$\Pi_1^{\rm QCD}(Q^2) = \frac{m_{N^*}}{2}\int_0^1 \frac{x_2^2 dx_2}{(q-px_2)^2}\int_0^{1-x_2} dx_1[3(1-t)[-A_3-V_3]+(1+t)[P_1+S_1+T_1-T_7]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}}{2}\int_0^1 \frac{x_3^2 dx_3}{(q-px_3)^2}\int_0^{1-x_3} dx_3[(1-t)[-A_3-V_3]+(1+t)[P_1+S_1+T_1-T_7]](x_1,1-x_1-x_3,x_3)$$
$$+\frac{m_{N^*}^3}{2}\int_0^1 \frac{x_2^2 dx_2}{(q-px_2)^4}\int_0^{1-x_2} dx_1[2(1-t)[2A_1^M+V_1^M]+3(1+t)T_1^M](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}}{2}\int_0^1 \frac{\alpha d\alpha}{(q-p\alpha)^2}\int_\alpha^1 dx_2\int_0^{1-x_2} dx_1[(1-t)[-3A_1+3A_2-3A_3+V_1-V_2-V_3]$$
$$+(1+t)[5T_1-T_2-3T_3-7T_7]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}}{2}\int_0^1 \frac{\alpha d\alpha}{(q-p\alpha)^2}\int_\alpha^1 dx_3\int_0^{1-x_3} dx_1[(1+t)[3T_1+T_2-2T_3-T_7]](x_1,1-x_1-x_3,x_3)$$
$$+\frac{m_{N^*}^3}{2}\int_0^1 \frac{\alpha^3 d\alpha}{(q-p\alpha)^4}\int_\alpha^1 dx_2\int_0^{1-x_2} dx_1[(1-t)[-3A_1+A_2-2A_3+A_4+2A_5+V_1-V_2-2V_3+V_4]$$

$$+(1+t)[P_1-P_2+S_1-S_2+2T_1-3T_2+2T_3-T_5-6T_7]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}^3}{2}\int_0^1\frac{\alpha^3 d\alpha}{(q-p\alpha)^4}\int_\alpha^1 dx_3\int_0^{1-x_3}dx_1[(1-t)[-A_1+A_2-2A_3+A_4+V_1-V_2-V_3+V_4]$$
$$+(1+t)[P_1-P_2+S_1-S_2+2T_1-3T_2+2T_3-T_5-6T_7]](x_1,1-x_1-x_3,x_3)$$
$$+\frac{m_{N^*}^3}{2}\int_0^1\frac{\beta^2 d\beta}{(q-p\beta)^4}\int_0^\beta d\alpha\int_\alpha^1 dx_2\int_0^{1-x_2}dx_1[2(1-t)[-2A_1+2A_2-2A_3-2A_4+2A_5-2A_6$$
$$+V_1-V_2-V_3-V_4-V_5+V_6]+(1+t)[7T_1-2T_2-5T_3-5T_4-2T_5+7T_6-9T_7-9T_8]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}^3}{2}\int_0^1\frac{\beta^2 d\beta}{(q-p\beta)^4}\int_0^\beta d\alpha\int_\alpha^1 dx_3\int_0^{1-x_3}dx_1[(1+t)[2T_1+T_2-3T_3-3T_4$$
$$+T_5+T_6-T_7-T_8]](x_1,1-x_1-x_3,x_3), \tag{B1}$$

$$\Pi_2^{\rm QCD}(Q^2)=-\frac{1}{2}\int_0^1\frac{1}{(q-px_2)^2}dx_2\int_0^{1-x_2}dx_1[(1-t)[A_1+V_1]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}^2}{2}\int_0^1\frac{1}{(q-px_2)^4}dx_2\int_0^{1-x_2}dx_1[3(1-t)[A_1^M+V_1^M]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}^2}{2}\int_0^1\frac{\alpha^2 d\alpha}{(q-p\alpha)^4}\int_\alpha^1 dx_2\int_0^{1-x_2}dx_1[(1-t)[-A_1-A_2-A_3+A_4+A_5-V_1+V_2+V_4]$$
$$+(1+t)[P_1-P_2+S_1-S_2+T_2-T_5]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}^2}{2}\int_0^1\frac{\alpha^2 d\alpha}{(q-p\alpha)^4}\int_\alpha^1 dx_3\int_0^{1-x_3}dx_1[(1-t)[A_1-A_2+A_4-V_1+V_2+V_4]$$
$$+(1+t)[P_1-P_2+S_1-S_2+T_2-T_5]](x_1,1-x_1-x_3,x_3)$$
$$+\frac{m_{N^*}^2}{2}\int_0^1\frac{\beta d\beta}{(q-p\beta)^4}\int_0^\beta d\alpha\int_\alpha^1 dx_2\int_0^{1-x_2}dx_1[(1+t)[T_2-T_3-T_4+T_5+T_7+T_8]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}^2}{2}\int_0^1\frac{\beta d\beta}{(q-p\beta)^4}\int_0^\beta d\alpha\int_\alpha^1 dx_3\int_0^{1-x_3}dx_1[(1+t)[T_2-T_3-T_4+T_5+T_7+T_8]](x_1,1-x_1-x_3,x_3), \tag{B2}$$

$$\Pi_3^{\rm QCD}(Q^2)=\frac{1}{2}\int_0^1 dx_2\frac{(1-x_2)}{(q-px_2)^2}\int_0^{1-x_2}[(1-t)[A_1+V_1]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{1}{2}\int_0^1 dx_3\frac{(1-x_3)}{(q-px_3)^2}\int_0^{1-x_3}[(1-t)[A_1-V_1]](x_1,1-x_1-x_3,x_3)$$
$$+\frac{1}{2}\int_0^1 dx_2\frac{(1-x_2)}{(q-px_2)^4}\int_0^{1-x_2}[(1-t)[3V_1^M+3A_1^M]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}^2}{2}\int_0^1\frac{d\alpha}{(q-p\alpha)^4}\int_\alpha^1 dx_2\int_0^{1-x_2}dx_1[(1-t)[-A_1-A_2-2A_3+A_4+2A_5-V_1+V_2+V_4]$$
$$+(1+t)[P_1-P_2+S_1-S_2+T_2-T_5]](x_1,x_2,1-x_1-x_2)$$
$$+\frac{m_{N^*}^2}{2}\int_0^1\frac{d\alpha}{(q-p\alpha)^4}\int_\alpha^1 dx_3\int_0^{1-x_3}dx_1[(1-t)[-A_1-A_2+A_4-V_1+V_2+V_4]$$
$$+(1+t)[P_1-P_2+S_1-S_2+T_2-T_5]](x_1,1-x_1-x_3,x_3))$$
$$+\frac{m_{N^*}^2}{2}\int_0^1\frac{\alpha(1-\alpha)}{(q-p\alpha)^4}d\alpha\int_\alpha^1 dx_2\int_0^{1-x_2}dx_1[2(1-t)[-A_1-A_2-2A_3+A_4+2A_5-V_1+V_2+V_4]$$
$$+(1+t)[P_1-P_2+S_1-S_2+T_2-T_5]](x_1,x_2,1-x_1-x_2)$$

$$
\begin{aligned}
&+\frac{m_{N^*}^2}{2}\int_0^1 \frac{\alpha(1-\alpha)}{(q-p\alpha)^4}d\alpha\int_\alpha^1 dx_3\int_0^{1-x_3}dx_1[(1-t)[A_1-A_2+A_4-V_1+V_2+V_4]\\
&+(1+t)[P_1-P_2+S_1-S_2-T_2-T_5]](x_1,1-x_1-x_3,x_3)\\
&+\frac{m_{N^*}^2}{2}\int_0^1 \frac{(1+\beta)}{(q-p\beta)^4}d\beta\int_0^\beta d\alpha\int_\alpha^1 dx_2\int_0^{1-x_2}dx_1(1+t)[T_2-T_3-T_4+T_5+T_7+T_8](x_1,x_2,1-x_1-x_2)\\
&+\frac{m_{N^*}^2}{2}\int_0^1 \frac{(1+\beta)}{(q-p\beta)^4}d\beta\int_0^\beta d\alpha\int_\alpha^1 dx_3\int_0^{1-x_3}dx_1[3(1+t)[T_2-T_3-T_4+T_5+T_7+T_8]](x_1,1-x_1-x_3,x_3),
\end{aligned}
\tag{B3}
$$

$$
\begin{aligned}
\Pi_4^{\rm QCD}(Q^2)=&\frac{m_{N^*}^3}{2}\int_0^1\frac{dx_2}{(p-px_2)^2}\int_0^{1-x_2}(1+t)T_1^M(x_1,x_2,1-x_1-x_2)\\
&+\frac{m_{N^*}^3}{8}\int_0^1\frac{\alpha d\alpha}{(q-p\alpha)^2}\int_\alpha^1 dx_2\int_0^{1-x_2}dx_1[(1-t)[-2A_1-2A_2-3A_3+A_4+4A_5-2V_1+2V_2-V_3+3V_4]\\
&+(1+t)[2P_1-2P_2+2S_1-2S_2-T_1+T_2+4T_3-4T_5-2T_7]](x_1,x_2,1-x_1-x_2)\\
&+\frac{m_{N^*}^3}{8}\int_0^1\frac{\alpha d\alpha}{(q-p\alpha)^2}\int_\alpha^1 dx_3\int_0^{1-x_3}dx_1[-2A_1+2A_2-3A_3+A_4+2V_1-2V_2-3V_3+V_4]\\
&+(1+t)[2P_1-2P_2+2S_1-2S_2+T_1-T_2-3T_5-T_7]](x_1,1-x_1-x_3,x_3)\\
&+\frac{m_{N^*}^3}{8}\int_0^1\frac{d\beta}{(q-p\beta)^2}\int_0^\beta d\alpha\int_\alpha^1 dx_2\int_0^{1-x_2}dx_1[4(1-t)[-A_1+A_2-A_3-A_4+A_5-A_6-V_1+V_2+V_3\\
&+V_4+V_5-V_6]+(1+t)[T_1-T_2-T_5+T_6-2T_7-2T_8]](x_1,x_2,1-x_1-x_2)\\
&+\frac{m_{N^*}^3}{8}\int_0^1\frac{d\beta}{(q-p\beta)^2}\int_0^\beta d\alpha\int_\alpha^1 dx_3\int_0^{1-x_3}dx_1[3(1+t)[T_1-T_2-T_5+T_6-2T_7-2T_8]](x_1,1-x_1-x_3,x_3).
\end{aligned}
\tag{B4}
$$

As we noticed previously, on the QCD side, we start the calculations in x-space, then transfer them to the momentum space by performing the corresponding Fourier integrals. To eliminate the unwanted contributions coming from the excited and continuum states in the correlation function, we carry out the Borel transformation. After the Borel transformation, the contribution of the unwanted terms is exponentially suppressed. We also apply the continuum subtraction procedure. The Borel transformation and continuum subtraction are performed using the subsequent rules [39]:

$$
\begin{aligned}
&\int dz\frac{\rho(z)}{\Theta^2}\to-\int_{x_0}^1\frac{dz}{z}\rho(z)e^{-s(z)/M^2},\\
&\int dz\frac{\rho(z)}{\Theta^4}\to\frac{1}{M^2}\int_{x_0}^1\frac{dx}{z^2}\rho(z)e^{-s(z)/M^2}+\frac{\rho(x_0)}{Q^2+x_0^2m_{N^*}^2}e^{-s_0/M^2},
\end{aligned}
$$

where

$$
\begin{aligned}
\Theta&=q-zp,\\
s(z)&=(1-z)m_{N^*}^2+\frac{1-z}{z}Q^2,\\
x_0&=\left(\sqrt{(Q^2+s_0-m_{N^*}^2)^2+4m_{N^*}^2Q^2}-(Q^2+s_0-m_{N^*}^2)\right)/2m_{N^*}^2.
\end{aligned}
$$

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