Gravitational form factors of the Δ baryon via QCD sum rules

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The gravitational form factors of a hadron are defined through the matrix elements of the energy-momentum tensor current, which can be decomposed into the quark and gluonic parts, between the hadronic states. These form factors provide important information for answering fundamental questions about the distribution of the energy, the spin, the pressure, and the shear forces inside the hadrons. Theoretical and experimental studies of these form factors provide exciting insights on the inner structure and geometric shapes of hadrons. Inspired by this, the gravitational form factors of Δ resonance are calculated by employing the QCD sum rule approach. The acquired gravitational form factors are used to calculate the composite gravitational form factors like the energy and angular momentum multipole form factors, D-terms related to the mechanical properties like the internal pressure and shear forces, as well as the mass radius of the system. The predictions are compared with the existing results in the literature.

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I. INTRODUCTION

The key subject of nonperturbative QCD is to figure out the inner structure of hadrons and their properties concerning the degrees of freedom of quarks and gluons. Different hadronic charges characterized as matrix elements of the vector, axial, and tensor currents between hadronic states contain precise information about the internal structure distributions of different physical quantities and geometric shapes of the hadrons. Besides the electromagnetic, axial, and tensor form factors of hadrons, the gravitational form factors (GFFs) or energy-momentum tensor form factors are also fundamental constituents to investigate the inner organizations of hadrons. These form factors give us a tool to systematically study the properties of the hadrons and to calculate several related observables such as spin, multipole form factors, mass and mechanical radii, shear force, and energy-pressure distributions inside the hadrons. Understanding the mechanical structure of hadrons is important because it gives us fundamental information about the internal structure and geometric shapes of hadrons as stated.

In recent years, GFFs have attracted increasing interests in describing the features of hadrons with different spins because of their relation to the generalized parton distributions (GPDs). The GPDs can be extracted from available data of hard exclusive processes like deeply virtual Compton scattering, deeply virtual meson production, wide-angle Compton scattering, single diffractive hard exclusive processes, and different vector-meson electroproduction processes. The GFFs can be directly calculated from the theory, as well. Comparison of GFFs calculated from pure theory with the ones extracted from the GPDs are indirect comparison of the experimental data with theoretical predictions on many physical observables. Such comparison for the GFFs of nucleon is made in Ref. [1]: The consistency of the results obtained from both sides show that the mankind is in the right way regarding the theoretical and experimental extractions of the nucleon properties.

The GFFs for the spin-$1/2$ particles have been parametrized in Refs. [2–5]. Utilizing these parametrizations, the GFFs of spin $1/2$ baryons have been studied in different phenomenological models [1,6–41]. For a spin-1 particle, the corresponding GFFs were studied in Refs. [29,42–49]. The GFFs for the spin-$3/2$ states have also been investigated in Refs. [29,50,50–57]. The computations of GFFs have also been extended to the $N^* \to N$ and $N \to \Delta$ transitions in Refs. [58–62]. To this end, methods like the lattice QCD, the light-cone QCD sum rule, the chiral

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effective theory, the chiral quark model, the $SU(2)$ Skyrme model, the AdS/CFT correspondence, and the bag model have been used.

In the present study, the GFFs of the Δ baryon are calculated utilizing the three-point QCD sum rule technique, as one of the powerful and successful nonperturbative methods in hadron physics. With the help of this method, we extract the behavior of the Δ baryon's GFFs with respect to Q^2 and, in connection with this, the mechanical properties of this resonance: The energy and angular momentum multipole form factors, $\mathcal{D}$ terms related to the internal pressure and shear forces as well as the mass radius. On the contrary to the electromagnetic form factors of the Δ baryon, which have been widely studied both theoretically and experimentally [50,63,64], it is quite difficult to extract the GFFs of the Δ baryon experimentally or obtain them from the corresponding GPDs due to the short-lived nature of the Δ baryon. The GFFs of the Δ baryon and the corresponding mechanical properties have theoretically been studied using the relativistic covariant quark-diquark approach [50], chiral EFT [51], lattice QCD (for the gluonic part) [29], and the $SU(2)$ Skyrme model based on the large N_c limit [55]. More systematic studies are needed to examine the features of Δ baryon GFFs. Recently, GPDs of spin-3/2 hadrons and the sum rules that connect the GPDs with the GFFs have explicitly been displayed in Refs. [57,65].

The remainder of this paper is structured as follows: In Sec. II, GFFs of Δ baryon calculated via the three-point QCD sum rules approach are introduced. The gravitational multipole form factors of the Δ baryon are given in Sec. III. Numerical analysis of the GFFs and the mechanical properties of the Δ baryon are presented in Sec. IV. In the last section, we conclude our work with a discussion of the obtained results.

II. QCD SUM RULES FOR THE GRAVITATIONAL FORM FACTORS OF THE Δ BARYON

We use the QCD sum rules to calculate the gravitational form factors of the Δ baryon. For this purpose, we consider the following three-point correlation function,

$$\Pi_{\alpha\mu\nu\beta}(p,q) = i^2 \int d^4x e^{-ip.x} \int d^4y e^{ip'.y} \times \langle 0|\mathcal{T}[J^{\Delta}_{\alpha}(y) T_{\mu\nu}(0) \bar{J}^{\Delta}_{\beta}(x)]|0\rangle, \quad (1)$$

where $\mathcal{T}$ denotes the time ordering operator, p (p') is the four-momentum of the initial (final) Δ baryon, $q = p - p'$ is the momentum transfer, $J^{\Delta}_{\alpha}(y)$ is the interpolating current for the Δ state at point y, and $T_{\mu\nu}$ is the energy-momentum tensor (EMT) current. The interpolating current for Δ^+ is given by

$$J_\alpha(x) = \frac{1}{\sqrt{3}} \varepsilon^{abc} \Big[2\Big(u^{aT}(x) C\gamma_\alpha d^b(x)\Big) u^c(x) + \Big(u^{aT}(x) C\gamma_\alpha u^b(x)\Big) d^c(x)\Big], \quad (2)$$

where C is the charge conjugation operator; and a, b, and c are color indices. The EMT current has two parts: One from the quarks and another one from the gluons, as given below,

$$T_{\mu\nu}(z) = T^q_{\mu\nu}(z) + T^g_{\mu\nu}(z), \quad (3)$$

with

$$T^q_{\mu\nu}(z) = \frac{i}{2}\Big[\bar{u}(z)\overleftrightarrow{D}_\mu\gamma_\nu u(z) + \bar{u}(z)\overleftrightarrow{D}_\nu\gamma_\mu u(z) + \bar{d}(z)\overleftrightarrow{D}_\mu\gamma_\nu d(z) + \bar{d}(z)\overleftrightarrow{D}_\nu\gamma_\mu d(z)\Big] - g_{\mu\nu}\Big[\bar{u}(z)\Big(i\overleftrightarrow{\not{D}} - m_u\Big)u(z) + \bar{d}(z)\Big(i\overleftrightarrow{\not{D}} - m_d\Big)d(z)\Big], \quad (4)$$

$$T^g_{\mu\nu}(z) = \frac{1}{4} g_{\mu\nu} G^{\rho\delta}(z) G_{\rho\delta}(z) - G_{\mu\rho}(z) G^{\rho}_{\nu}(z). \quad (5)$$

We can rewrite the second term of the quark EMT current in Eq. (4) as follows [66],

$$g_{\mu\nu}\Big[\bar{u}(z)\Big(i\overleftrightarrow{\not{D}} - m_u\Big)u(z) + \bar{d}(z)\Big(i\overleftrightarrow{\not{D}} - m_d\Big)d(z)\Big] \simeq g_{\mu\nu}(1+\gamma_m)\Big(m_u\bar{u}u + m_d\bar{d}d\Big), \quad (6)$$

where γ_m denotes the anomalous dimension of the mass operator. We assume the chiral limit where $m_u = m_d = 0$. This eliminates the term in Eq. (6). The covariant derivative $\overleftrightarrow{D}_\mu$ in Eq. (4) is given by

$$\overleftrightarrow{D}_\mu(z) = \frac{1}{2}[\overrightarrow{D}_\mu(z) - \overleftarrow{D}_\mu(z)], \quad (7)$$

with

$$\overrightarrow{D}_\mu(z) = \overrightarrow{\partial}_\mu(z) - i\frac{g}{2}\lambda^a A^a_\mu(z),$$
$$\overleftarrow{D}_\mu(z) = \overleftarrow{\partial}_\mu(z) + i\frac{g}{2}\lambda^a A^a_\mu(z), \quad (8)$$

where, λ^a are the Gell-Mann matrices and $A^a_\mu(z)$ are the external gluon fields. Using the Fock-Schwinger gauge, $z^\mu A^a_\mu(z) = 0$, the gluon fields can be expressed in terms of the gluon field strength tensor by

$$A^a_\mu(z) = \int_0^1 d\alpha \alpha z_\xi G^a_{\xi\mu}(\alpha z) = \frac{1}{2} z_\xi G^a_{\xi\mu}(0) + \frac{1}{3} z_\eta z_\xi D_\eta G^a_{\xi\mu}(0) + \cdots. \quad (9)$$

To calculate the derivative terms of the quark part of the EMT current, we evaluate the EMT current at point z in Eq. (1) and finally take the limit $z \to 0$. In this limit, Eq. (9) shows that the gluon field vanishes and therefore the covariant derivatives in Eq. (8) become partial derivatives.

In the QCD sum rule approach, we define the correlation function in two different representations: One based on the hadronic degrees of freedom and is called the physical (phenomenological) side and the other based on QCD degrees of freedom and is called the QCD (theoretical) side. The double Borel transformations with respect to the momentum squared of the initial and final states are applied to both sides to remove/suppress the contributions coming from the subtraction terms/higher states and continuum. A continuum subtraction procedure supplied by quark-hadron duality assumption is also applied to further suppress the contributions of the higher states and enhance the ground state contribution. The form factors are obtained by matching the coefficients of the same Lorentz structures of both representations.

A. Physical side of the correlation function

We start by evaluating the correlation function in Eq. (1) using hadronic parameters. For this purpose, we insert two complete sets of the intermediate states $\Delta(p', s')$ and $\Delta(p, s)$ into Eq. (1) and perform the four-integrals over x and y, which ends up in

$$\Pi^{\rm Had}_{\alpha\mu\nu\beta}(p,q) = \sum_{s'} \sum_{s} \frac{\langle 0|J^\Delta_\alpha|\Delta(p',s')\rangle \langle \Delta(p',s')|T_{\mu\nu}(0)|\Delta(p,s)\rangle \langle \Delta(p,s)|\bar{J}^\Delta_\beta|0\rangle}{(m^2-p'^2)(m^2-p^2)} + \cdots, \quad (10)$$

where $m = m_\Delta$ and the dots indicate the higher states and continuum contributions. The matrix element of the EMT current between Δ states can be expressed in terms of ten form factors [54,55]

$$\begin{aligned}\langle \Delta(p',s')|T_{\mu\nu}(0)|\Delta(p,s)\rangle = -\bar{u}_{\alpha'}(p',s')\Bigg\{ & \frac{P_\mu P_\nu}{m}\left(g^{\alpha'\beta'}F_{1,0}(Q^2) - \frac{\Delta^{\alpha'}\Delta^{\beta'}}{2m^2}F_{1,1}(Q^2)\right) \\ & + \frac{(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)}{4m}\left(g^{\alpha'\beta'}F_{2,0}(Q^2) - \frac{\Delta^{\alpha'}\Delta^{\beta'}}{2m^2}F_{2,1}(Q^2)\right) + m g_{\mu\nu}\left(g^{\alpha'\beta'}F_{3,0}(Q^2) - \frac{\Delta^{\alpha'}\Delta^{\beta'}}{2m^2}F_{3,1}(Q^2)\right) \\ & + \frac{i}{2}\frac{(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{m}\left(g^{\alpha'\beta'}F_{4,0}(Q^2) - \frac{\Delta^{\alpha'}\Delta^{\beta'}}{2m^2}F_{4,1}(Q^2)\right) - \frac{1}{m}\Big(g^{\alpha'}_\mu \Delta_\nu \Delta^{\beta'} + g^{\alpha'}_\nu \Delta_\mu \Delta^{\beta'} + g^{\beta'}_\mu \Delta_\nu \Delta^{\alpha'} \\ & + g^{\beta'}_\nu \Delta_\mu \Delta^{\alpha'} - 2 g_{\mu\nu}\Delta^{\alpha'}\Delta^{\beta'} - \Delta^2 g^{\alpha'}_\mu g^{\beta'}_\nu - \Delta^2 g^{\alpha'}_\nu g^{\beta'}_\mu\Big)F_{5,0}(Q^2) + m\left(g^{\alpha'}_\mu g^{\beta'}_\nu + g^{\alpha'}_\nu g^{\beta'}_\mu\right)F_{6,0}(Q^2)\Bigg\} u_{\beta'}(p,s),\end{aligned} \quad (11)$$

where $u_{\beta'}(p,s)$ is the Rarita-Schwinger spinor with momentum p and spin s, $P = (p+p')/2$, $\Delta = p' - p$, $Q^2 = -\Delta^2$, and $F_{i,k}$ are GFFs. We consider the full system that includes both the quark and gluon contributions to the EMT, given by Eqs. (4) and (5), implying the conservation of the total current. Therefore, the nonconserved form factors $F_{i,k}(i = 3, 6)$ vanish, while the conserved ones $F_{i,k}(i = 1, 2, 4, 5)$ remain alive. By using the residue of the Δ baryon (λ_Δ), one can define the following matrix element:

$$\langle 0|J^\Delta_\alpha|\Delta(p',s')\rangle = \lambda_\Delta u_\alpha(p',s'). \quad (12)$$

We introduce the spin summation of the Rarita-Schwinger spinor for the Δ baryon as below,

$$\sum_{s'} u_\alpha(p',s')\bar{u}_{\alpha'}(p',s') = -(\not{p}' + m)\left[g_{\alpha\alpha'} - \frac{\gamma_\alpha\gamma_{\alpha'}}{3} - \frac{2p'_\alpha p'_{\alpha'}}{3m^2} + \frac{p'_\alpha\gamma_{\alpha'} - p'_{\alpha'}\gamma_\alpha}{3m}\right]. \quad (13)$$

Using Eqs. (11)–(13) in Eq. (10), we derive the following expression for the $\Delta \to \Delta$ transition three-point correlation function:

$$\Pi^{\rm Had}_{\alpha\mu\nu\beta}(p,q)=\frac{-\lambda_\Delta^2}{(m^2-p'^2)(m^2-p^2)}(\not{p}'+m)\left[g_{\alpha\alpha'}-\frac{\gamma_\alpha\gamma_{\alpha'}}{3}-\frac{2p'_\alpha p'_{\alpha'}}{3m^2}+\frac{p'_\alpha\gamma_{\alpha'}-p'_{\alpha'}\gamma_\alpha}{3m}\right]$$
$$\times\Bigg\{\frac{P_\mu P_\nu}{m}\left(g^{\alpha'\beta'}F_{1,0}(Q^2)-\frac{\Delta^{\alpha'}\Delta^{\beta'}}{2m^2}F_{1,1}(Q^2)\right)+\frac{(\Delta_\mu\Delta_\nu-g_{\mu\nu}\Delta^2)}{4m}\left(g^{\alpha'\beta'}F_{2,0}(Q^2)-\frac{\Delta^{\alpha'}\Delta^{\beta'}}{2m^2}F_{2,1}(Q^2)\right)$$
$$+\frac{i}{2}\frac{(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{m}\left(g^{\alpha'\beta'}F_{4,0}(Q^2)-\frac{\Delta^{\alpha'}\Delta^{\beta'}}{2m^2}F_{4,1}(Q^2)\right)-\frac{1}{m}\Big(g^{\alpha'}_\mu\Delta_\nu\Delta^{\beta'}+g^{\alpha'}_\nu\Delta_\mu\Delta^{\beta'}$$
$$+g^{\beta'}_\mu\Delta_\nu\Delta^{\alpha'}+g^{\beta'}_\nu\Delta_\mu\Delta^{\alpha'}-2g_{\mu\nu}\Delta^{\alpha'}\Delta^{\beta'}-\Delta^2 g^{\alpha'}_\mu g^{\beta'}_\nu-\Delta^2 g^{\alpha'}_\nu g^{\beta'}_\mu\Big)F_{5,0}(Q^2)\Bigg\}$$
$$\times(\not{p}+m)\left[g_{\beta'\beta}-\frac{\gamma_{\beta'}\gamma_\beta}{3}-\frac{2p_{\beta'}p_\beta}{3m^2}+\frac{p_{\beta'}\gamma_\beta-p_\beta\gamma_{\beta'}}{3m}\right]+\cdots. \tag{14}$$

In principle, the physical side of the correlation function can be obtained using the above equation. However, at this point we are faced with two problems that prevent the calculations being reliable: All Lorentz structures are not independent and the correlation function can also receive contributions from spin-1/2 particles, which should be eliminated. Indeed, the matrix element of the current J_α between vacuum and spin-1/2 baryons is nonzero and is determined as

$$\langle 0|J_\alpha(0)|B(p,s=1/2)\rangle=(Ap_\alpha+B\gamma_\alpha)u(p,s=1/2). \tag{15}$$

As is seen the unwanted spin-1/2 contributions are proportional to γ_α and p_α. By multiplying both sides with γ^α and employing the condition $\gamma^\alpha J_\alpha=0$ one can specify the constant A in terms of B. To eliminate the spin-1/2 contributions and acquire only independent structures in the correlation function, we use the ordering for Dirac matrices as $\gamma_\alpha\not{p}'\not{p}\gamma_\mu\gamma_\nu\gamma_\beta$ and remove terms with γ_α at the beginning, γ_β at the end, and those proportional to p'_α and p_β. After all manipulations mentioned above are done, we get the final form of the physical side of the correlation function as follows:

$$\Pi^{\rm Had}_{\alpha\mu\nu\beta}(Q^2)=\lambda_\Delta^2 e^{-\frac{m^2}{M^2}}\Big[\Pi^{\rm Had}_1(Q^2)p_\alpha p_\mu p_\nu p'_\beta\not{p}'+\Pi^{\rm Had}_2(Q^2)p_\alpha p_\mu p'_\nu p'_\beta\not{p}+\Pi^{\rm Had}_3(Q^2)p_\alpha p'_\mu p'_\nu p'_\beta\not{p}+\Pi^{\rm Had}_4(Q^2)p_\mu p_\nu g_{\alpha\beta}\not{p}$$
$$+\Pi^{\rm Had}_5(Q^2)p_\mu p'_\nu g_{\alpha\beta}\not{p}+\Pi^{\rm Had}_6(Q^2)p'_\mu p'_\nu g_{\alpha\beta}\not{p}+\Pi^{\rm Had}_7(Q^2)p'_\beta p'_\nu g_{\alpha\mu}\not{p}+\cdots\Big], \tag{16}$$

where the double Borel transformation on the variables p^2 and p'^2 with Borel parameter M^2 is applied. The initial and final states of the process involve Δ baryons, which have the same Borel mass parameter $M_i^2=M_f^2=2M^2$. The functions $\Pi^{\rm Had}_i(Q^2)$ are functions of the GFFs and other hadronic parameters. We kept only the Lorentz structures that we use to calculate the conserved GFFs and moved the others inside the dots.

B. QCD side of the correlation function

Having the expression of the correlation function from the physical side, let us turn our attention to the evaluation of the correlation function from the QCD side. To this end, we need to insert the explicit forms of the EMT current and interpolating current of the Δ baryon into the correlation function. Substituting Δ's interpolating current and the EMT current of Eqs. (4) and (5) into the three-point correlation function of Eq. (1), we get

$$\Pi^{\rm QCD}_{\alpha\mu\nu\beta}(p,q)=\frac{i^2}{6}\varepsilon^{abc}\varepsilon^{a'b'c'}\int d^4xe^{-ip.x}\times\int d^4ye^{ip'.y}\left(\Gamma^q_{\alpha\mu\nu\beta}+\Gamma^g_{\alpha\mu\nu\beta}\right). \tag{17}$$

The quark and gluon contributions of the EMT current yield Γ^q and Γ^g, respectively. Using Wick's theorem, Γ^q and Γ^g are obtained in terms of the quark propagators. The expressions for Γ^q and Γ^g are too long to show here, so we refer to Eqs. (A1) and (A2) in Appendix A.

Substituting the light quark propagators in Eqs. (A1) and (A2) and employing covariant derivatives of Eq. (7) and then considering $z\to 0$, we get

$$\Pi^{\rm QCD}_{\alpha\mu\nu\beta}(p,q)=\int d^4xe^{-ip.x}\int d^4ye^{ip'.y}\Gamma_{\alpha\mu\nu\beta}(x,y), \tag{18}$$

with

$$\Gamma_{\alpha\mu\nu\beta}(x,y)=\Big\{\Gamma^{(P)}_{\alpha\mu\nu\beta}+\Gamma^{(3D)}_{\alpha\mu\nu\beta}+\Gamma^{(4D,q)}_{\alpha\mu\nu\beta}+\Gamma^{(5D)}_{\alpha\mu\nu\beta}+\mu\leftrightarrow\nu\Big\}+\Gamma^{(4D,g)}_{\alpha\mu\nu\beta}, \tag{19}$$

where the correlation function has a perturbative part $\Gamma^{(P)}$ and nonperturbative parts $\Gamma^{(3D)}$, $\Gamma^{(4D)}$, and $\Gamma^{(5D)}$ in three, four, and five dimensions, respectively, which are shown in Appendix A. The four-dimensional nonperturbative parts for quark and gluon, $\Gamma^{(4D,q)}$ and $\Gamma^{(4D,g)}$, involve the products of two gluon field strength tensors $G^A_{\alpha\beta}$, which lead to gluon condensation as explained in Appendix B.

We transform the calculations to momentum space, by employing [67]

$$\frac{1}{(R^2)^{n_j}}=\int\frac{d^Dk_j}{(2\pi)^D}e^{-ik_j.R}i(-1)^{n_j+1}2^{D-2n_j}\pi^{D/2}\times\frac{\Gamma[D/2-n_j]}{\Gamma[n_j]}\left(\frac{-1}{k_j^2}\right)^{D/2-n_j}, \tag{20}$$

where $R=x$, y, or $y-x$ and we set $x_\mu=i\partial/\partial p_\mu$ and $y_\mu=-i\partial/\partial p'_\mu$. The integrals over x and y in D dimensions produce two Dirac Delta functions and simplify two of the D-dimensional integrals over k_j. The final integral takes simple forms after Feynman parametrizations. To perform them, we apply the general formula presented in [67], which takes the following form in the simplest case:

$$\int d^D\ell\frac{1}{(\ell^2+L)^n}=\frac{i\pi^{D/2}(-1)^n\Gamma[n-D/2]}{\Gamma[n](-L)^{n-D/2}}. \tag{21}$$

Following these calculations, the QCD side of the correlation function is derived as the double dispersion integrals shown below,

$$\Pi_i^{\rm QCD}(Q^2)=\int_0^{s_0}ds\int_0^{s_0}ds'\frac{\rho_i(s,s',Q^2)}{(s-p^2)(s'-p'^2)}, \tag{22}$$

where s_0 is the continuum. The imaginary parts of the $\Pi_i^{\rm QCD}(Q^2)$ define the spectral densities $\rho_i(s,s',Q^2)$, such that $\rho_i(s,s',Q^2)=\mathrm{Im}[\Pi_i^{\rm QCD}(Q^2)]/\pi$. To determine the imaginary parts of different structures, we use

$$\Gamma[D/2-n]\left(\frac{-1}{L}\right)^{D/2-n}=\frac{(-1)^{n-1}}{(n-2)!}(-L)^{n-2}\ln[-L]. \tag{23}$$

The expressions for the spectral densities $\rho_i(s,s',Q^2)$ are very lengthy and, for the sake of simplicity, we do not present them explicitly.

In parallel to the physical side, we consider the same ordering for Dirac matrices and the procedure for the elimination of the spin-1/2 pollution. We apply the double Borel transformation to the QCD side and obtain,

$$\begin{aligned}\Pi^{\rm QCD}_{\alpha\mu\nu\beta}(Q^2)=\int_0^{s_0}ds\int_0^{s_0}ds'e^{-s/2M^2}e^{-s'/2M^2}\Big[&\Pi_1^{\rm QCD}(Q^2,s,s')p_\alpha p_\mu p_\nu p'_\beta\not{p}+\Pi_2^{\rm QCD}(Q^2,s,s')p_\alpha p_\mu p'_\nu p'_\beta\not{p}\\&+\Pi_3^{\rm QCD}(Q^2,s,s')p_\alpha p'_\mu p'_\nu p'_\beta\not{p}+\Pi_4^{\rm QCD}(Q^2,s,s')p_\mu p_\nu g_{\alpha\beta}\not{p}+\Pi_5^{\rm QCD}(Q^2,s,s')p_\mu p'_\nu g_{\alpha\beta}\not{p}\\&+\Pi_6^{\rm QCD}(Q^2,s,s')p'_\mu p'_\nu g_{\alpha\beta}\not{p}+\Pi_7^{\rm QCD}(Q^2,s,s')p'_\beta p'_\nu g_{\alpha\mu}\not{p}+\cdots\Big],\end{aligned} \tag{24}$$

in terms of the selected structures. Matching the same structures from the QCD and physical sides, the GFFs for the Δ baryon are derived. We again do not show the obtained sum rules in this step.

III. GRAVITATIONAL MULTIPOLE FORM FACTORS

Having determined the seven conserved GFFs for the $\Delta\to\Delta$ gravitonlike transition we can now define some composite observables in terms of GFFs. Such observables provide with us useful information about the inner structure, distributions of different charges, and geometric shape of the hadron under consideration. Future experiments may provide opportunity for such observales to be measured. Hence, we provide sum inputs to be compared with possible related future experimental data. To this end, we use the following definitions for the kinematical variables P^μ, Δ^μ, and momentum transfer squared Q^2 in the Breit frame,

$$P^\mu=(E,\vec{0}),\quad \Delta^\mu=(0,\vec{\Delta}),\quad Q^2=-\Delta^2=4(E^2-m^2). \tag{25}$$

In this frame, we can express the gravitational multipole form factors (GMFFs) of the Δ baryon in terms of the conserved GFFs as follows [55]:

$$\varepsilon_0(Q^2) = F_{1,0}(Q^2) - \frac{Q^2}{6m^2}\left[-\frac{5}{2}F_{1,0}(Q^2) - F_{1,1}(Q^2) - \frac{3}{2}F_{2,0}(Q^2) + 4F_{5,0}(Q^2) + 3F_{4,0}(Q^2)\right] + \frac{(Q^2)^2}{12m^4}\left[\frac{1}{2}F_{1,0}(Q^2) + F_{1,1}(Q^2) + \frac{1}{2}F_{2,0}(Q^2) + \frac{1}{2}F_{2,1}(Q^2) - 4F_{5,0}(Q^2) - F_{4,0}(Q^2) - F_{4,1}(Q^2)\right] + \frac{(Q^2)^3}{48m^6}\left[-\frac{1}{2}F_{1,1}(Q^2) - \frac{1}{2}F_{2,1}(Q^2) + F_{4,1}(Q^2)\right], \tag{26}$$

$$\varepsilon_2(Q^2) = -\frac{1}{6}[F_{1,0}(Q^2) + F_{1,1}(Q^2) - 4F_{5,0}(Q^2)] + \frac{Q^2}{12m^2}\left[\frac{1}{2}F_{1,0}(Q^2) + F_{1,1}(Q^2) + \frac{1}{2}F_{2,0}(Q^2) + \frac{1}{2}F_{2,1}(Q^2) - 4F_{5,0}(Q^2) - F_{4,0}(Q^2) - F_{4,1}(Q^2)\right] + \frac{(Q^2)^2}{48m^4}\left[-\frac{1}{2}F_{1,1}(Q^2) - \frac{1}{2}F_{2,1}(Q^2) + F_{4,1}(Q^2)\right], \tag{27}$$

$$\mathcal{J}_1(Q^2) = \frac{1}{3}F_{4,0}(Q^2) - \frac{Q^2}{15m^2}\left[F_{4,0}(Q^2) + F_{4,1}(Q^2) + 5F_{5,0}(Q^2)\right] + \frac{(Q^2)^2}{60m^4}F_{4,1}(Q^2), \tag{28}$$

$$\mathcal{J}_3(Q^2) = -\frac{1}{6}\left[F_{4,0}(Q^2) + F_{4,1}(Q^2)\right] + \frac{Q^2}{24m^2}F_{4,1}(Q^2), \tag{29}$$

$$D_0(Q^2) = F_{2,0}(Q^2) - \frac{16}{3}F_{5,0}(Q^2) - \frac{Q^2}{6m^2}\left[F_{2,0}(Q^2) + F_{2,1}(Q^2) - 4F_{5,0}(Q^2)\right] + \frac{(Q^2)^2}{24m^4}F_{2,1}(Q^2), \tag{30}$$

$$D_2(Q^2) = \frac{4}{3}F_{5,0}(Q^2), \tag{31}$$

$$D_3(Q^2) = \frac{1}{6}\left[-F_{2,0}(Q^2) - F_{2,1}(Q^2) + 4F_{5,0}(Q^2)\right] + \frac{Q^2}{24m^2}F_{2,1}(Q^2), \tag{32}$$

where $\varepsilon_0(Q^2)$, $\varepsilon_2(Q^2)$, $\mathcal{J}_1(Q^2)$, and $\mathcal{J}_3(Q^2)$ are energy-monopole, energy-quadrupole, angular momentum-dipole, and angular momentum-octupole form factors, respectively. The form factors $D_{0,2,3}(Q^2)$ are related to the internal pressures and shear forces [66]. These form factors are used to define the generalized D-terms $\mathcal{D}_{0,2,3}$ of Δ baryon in the following way [53]:

$$\mathcal{D}_0 = D_0(0),$$
$$\mathcal{D}_2 = D_2(0) + \frac{2}{m^2}\int_0^\infty dQ^2 D_3(Q^2),$$
$$\mathcal{D}_3 = -\frac{5}{m^2}\int_0^\infty dQ^2 D_3(Q^2). \tag{33}$$

The generalized D-terms are dimensionless quantities that characterize the elastic properties of hadrons. The mean square radius of the energy density, also known as the mass radius, is another important mechanical property of the Δ baryon. It is given by the following formula [55,66]:

$$\langle r_E^2\rangle = 6\frac{d\varepsilon_0(k)}{dk}\bigg|_{k=0}. \tag{34}$$

In the following section, we will perform numerical analysis of the obtained GFFs and other observables made of these GFFs and discuss their values at zero momentum transfer.

IV. NUMERICAL RESULTS

In this section, we numerically analyze the form factors derived from the sum rules in the previous sections. The values of some input parameters are given as $m_u = m_d = 0$, $m_\Delta = 1.23$ GeV, $\lambda_\Delta = 0.038$ GeV^3 [68], $\langle\bar{q}q\rangle(1\ \text{GeV}) = (-0.24 \pm 0.01)^3$ GeV^3 [69], $m_0^2 = (0.8 \pm 0.1)$ GeV^2 [69], $\langle\frac{\alpha_s}{\pi}G^2\rangle = (0.012 \pm 0.004)$ GeV^4 [70], and $\alpha_s = (0.118 \pm 0.005)$ [71]. In addition to these input parameters, there are two more auxiliary parameters called the Borel parameter M^2 and the continuum threshold s_0 that we use for the sum rules. According to the philosophy of the QCD sum rules, these auxiliary parameters should not affect the physical quantities. However, in practice, it is not possible to provide such a situation. Therefore, we look for working regions where the GFFs have weak dependence on these helping parameters. The residual dependencies appear as the uncertainties in the final results. The continuum threshold s_0 is associated with the energy of the first exited state. To restrict the Borel parameter, we require the pole dominance and convergence of the operator product expansion: The perturbative contribution exceeds the total nonperturbative one and the higher the dimension of the nonperturbative operator the lower its contribution. Our calculations reveal the following working regions for the s_0 and M^2:

$$2.9\ \text{GeV}^2 \le s_0 \le 3.3\ \text{GeV}^2, \qquad 2.0\ \text{GeV}^2 \le M^2 \le 3.0\ \text{GeV}^2. \tag{35}$$

In Fig. 1, we present the Borel mass parameter dependence of the GFFs at $Q^2 = 1.0\ \text{GeV}^2$ and three values of the continuum threshold $s_0 = 2.9, 3.1$, and $3.3\ \text{GeV}^2$. This figure shows that the GFFs are stable with respect to the change of Borel mass parameter in the working region. In Fig. 2, we present the GFFs as a function of Q^2 for the fixed Borel mass $M^2 = 2.5\ \text{GeV}^2$ and three values of the continuum threshold $s_0 = 2.9, 3.1$, and $3.3\ \text{GeV}^2$. As expected, we observe that the Q^2 dependencies of the GFFs are smoothly changed and decrease with increasing the Q^2. We use the following p-pole fit function to fit the GFFs from the sum rules predictions:

$$\mathcal{F}(Q^2) = \frac{\mathcal{F}(0)}{(1 + m_p Q^2)^p}, \tag{36}$$

where the fit parameters $\mathcal{F}(0)$ and p are dimensionless and m_p has the inverse square energy dimension. The p-pole fit function of Δ's GFFs tends to zero at large $Q^2 = 10\ \text{GeV}^2$, as Fig. 2 illustrates. To enhance the visibility, Fig. 3 shows the Q^2 dependence of Δ's GFFs for $0\ \text{GeV}^2 \le Q^2 \le 2\ \text{GeV}^2$. The p-pole fit parameters of the GFFs in Fig. 2 at mean values of the continuum threshold are summarized in Table I. Changes in the working regions of auxiliary parameters, uncertainty in the input parameters, as well as the systematic errors in QCD sum rules method cause errors in our presented results. Some mechanical properties are revealed by the Δ's GFFs at zero momentum transfer, which are shown in the second column of this table as $\mathcal{F}(0)$.

We present and compare some mechanical properties extracted from our work and other studies in the rest of this section. Table II shows some of the GMFFs of Δ baryon at zero momentum transfer obtained from our calculations and compares them with the results of Ref. [55]. The normalization condition for Δ mass is 1, which is consistent with $\varepsilon_0(0) = F_{1,0}(0) = 1.01 \pm 0.15$ from our calculations. We obtain $\mathcal{J}_1(0) = \frac{1}{3} F_{4,0}(0) = 0.46 \pm 0.06$ for the dipole angular momentum where $F_{4,0}(0) = 1.38 \pm 0.19$ corresponds to spin of Δ baryon which is $3/2$. This result is well consistent with the prediction of the Skyrme model within the presented errors. We obtain a p-pole behavior for the octupole angular momentum form factor $\mathcal{J}_3(Q^2)$ from our calculations and Eq. (29). At zero momentum transfer, we have $\mathcal{J}_3(0) = -\frac{1}{6}[F_{4,0}(0) + F_{4,1}(0)] = -0.17 \pm 0.03$ and at large momentum transfer $Q^2 = 10$, $\mathcal{J}_3(Q^2)$ approach to zero. In contrast, Ref. [55] assumes that $\mathcal{J}_3(Q^2)$ is zero for all values of Q^2 to suppress the corresponding density in the large N_c expansion. Our obtained $\varepsilon_2(0) = -0.18 \pm 0.03$ differs from the corresponding value in Ref. [55].

Except for $F_{1,1}(0)$ and $F_{4,1}(0)$, our results for Δ's GFFs at zero momentum transfer, $\mathcal{F}(0)$ in Table I, are comparable with the corresponding results in Ref. [55]. We obtain $F_{1,1}(0) = -0.42 \pm 0.05$ and $F_{4,1}(0) = -0.35 \pm 0.03$ from our calculations, which contrast with $F_{1,1}(0) = -3.64$ and $F_{4,1}(0) = -1.5$ in Ref. [55]. By applying the Skyrme model with the constraints $\varepsilon_0(0) = F_{1,0}(0) = 1$ and $\mathcal{J}_1(0) = \frac{1}{3} F_{4,0}(0) = \frac{1}{2}$ and the assumption $\mathcal{J}_3(0) = -\frac{1}{6}[F_{4,0}(0) + F_{4,1}(0)] = 0$, Ref. [55] obtains $F_{4,1}(0) = -F_{4,0}(0) = -1.5$. The sum rules method allows us to define Δ's GFFs without imposing any additional conditions on GFFs and GMFFs, which is an advantage of this method. The ratio $F_{1,1}(0)/F_{4,1}(0)$ is obtained using our sum rules and compared with some other models' predictions [51,55], as shown by

$$\frac{F_{1,1}(0)}{F_{4,1}(0)} \simeq \begin{cases} 2 & \text{tree-level chiral perturbation theory (ChPT)}, \\ 2.43 & \text{Skyrme model}, \\ 1.2 \pm 0.25 & \text{current work}. \end{cases} \tag{37}$$

As is seen, the different approaches agree on the sign this ratio and the obtained magnitudes are roughly close to each other. Note that it is not possible to extract the values for $F_{1,1}(0)$ and $F_{4,1}(0)$ using ChPT, because the needed coupling constants are not fixed (see Ref. [51]).

Our results for $D_2(0)$ and $D_3(0)$ agree with those of Ref. [55] while for $D_0(0)$ differs from the corresponding value in this reference considerably. The D-terms and the mass radius of our calculations for the Δ baryon are shown in Table III along with the predictions of other models. We get $\langle r_E^2 \rangle = 0.67 \pm 0.04\ \text{fm}^2$ for the mass radius from Eq. (34), which agrees, within the uncertainties of our result, with the $0.64\ \text{fm}^2$ reported in Refs. [52,55]. While our result for $\mathcal{D}_0^\Delta$ is consistent with that of Ref. [53], it is quite different from the prediction of the

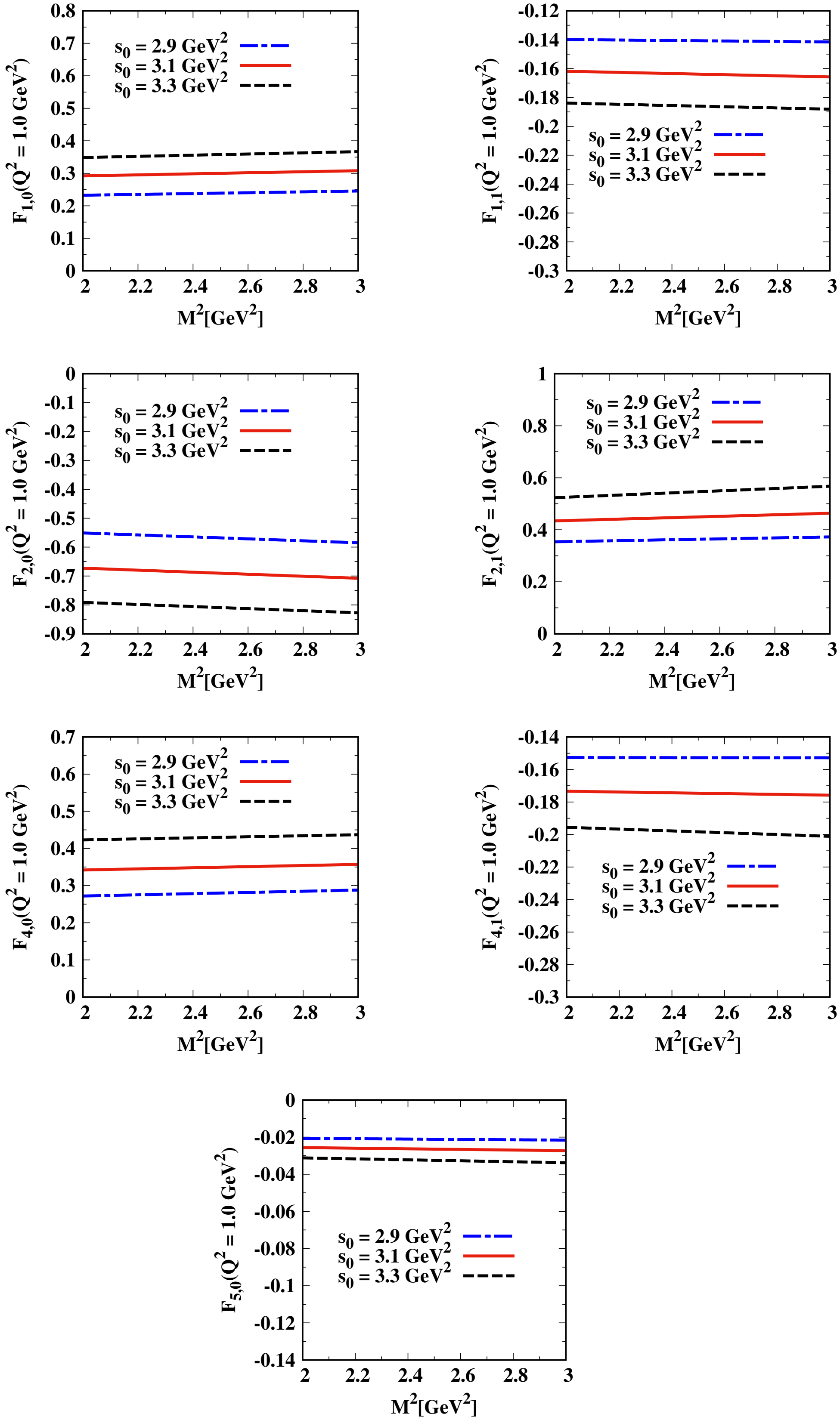

FIG. 1. The dependence of the GFFs of Δ on M^2 at $Q^2 = 1.0$ GeV^2 for three values of the continuum threshold s_0.

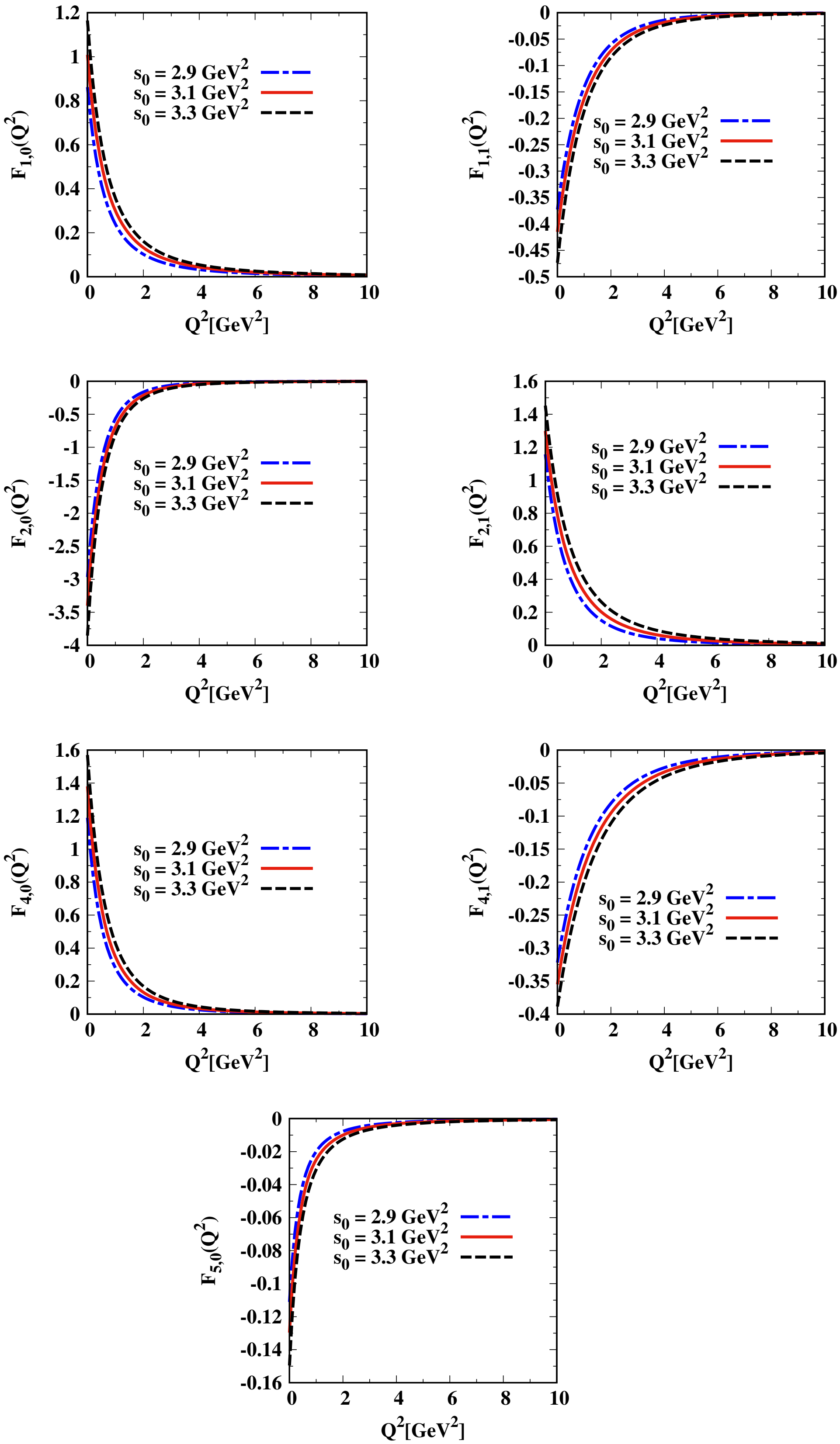

FIG. 2. The dependence of the GFFs of Δ on Q^2 at $M^2 = 2.5\ \mathrm{GeV}^2$ for three values of s_0.

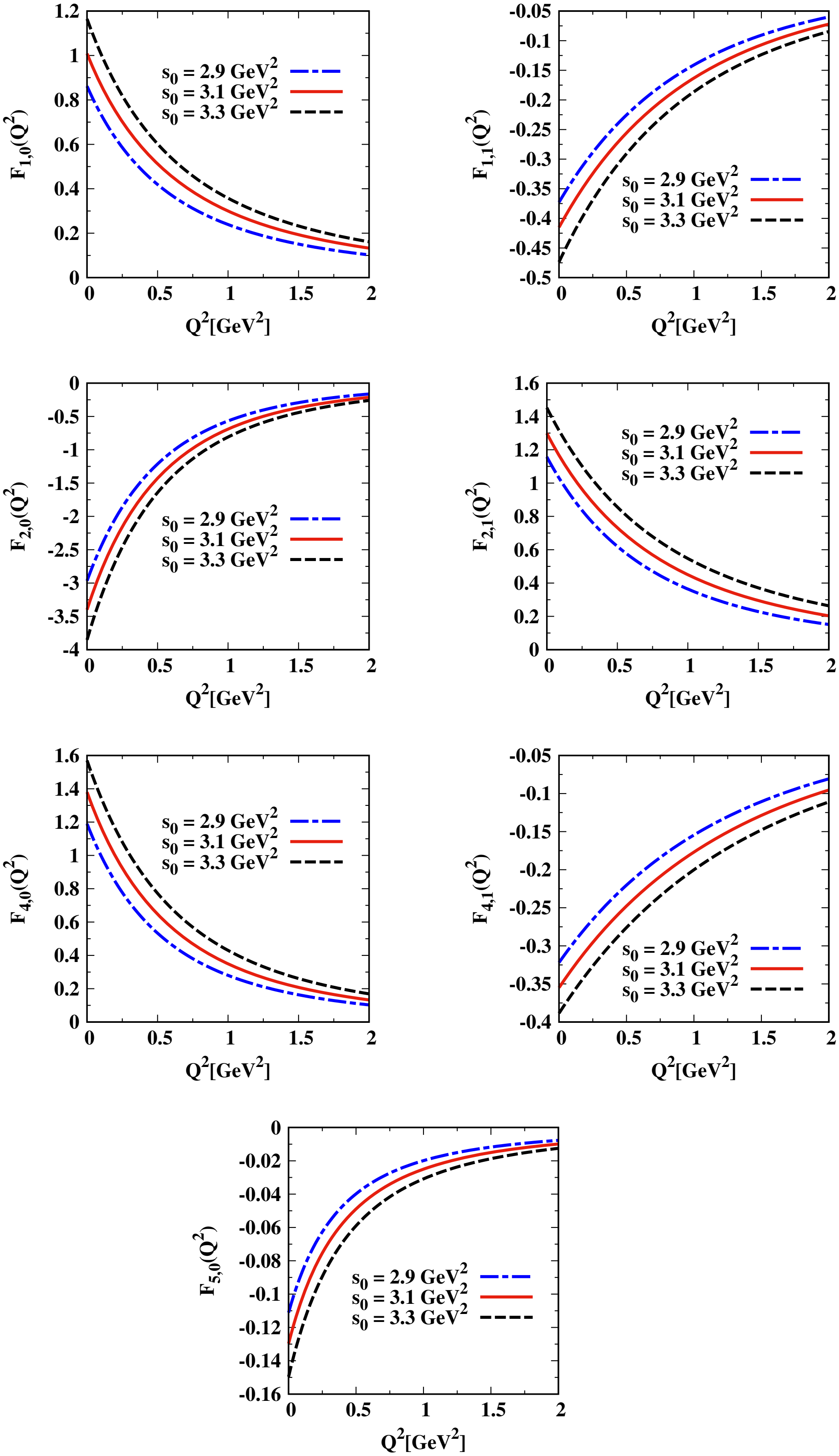

FIG. 3. The Q^2 dependence of Δ's GFFs at $M^2 = 2.5$ GeV^2 for three values of s_0, where we restricted 0 GeV$^2 \leq Q^2 \leq 2$ GeV^2 for more clarity.

TABLE I. The numerical values of p-pole fit parameters $\mathcal{F}(0)$, m_p, and p of the GFFs in Fig. 2 at mean values of the continuum threshold.

GFF	$\mathcal{F}(0)$	m_p (GeV^{-2})	p
$F_{1,0}(Q^2)$	1.01 ± 0.15	0.63 ± 0.03	2.52 ± 0.04
$F_{1,1}(Q^2)$	-0.42 ± 0.05	0.17 ± 0.03	6.23 ± 1.08
$F_{2,0}(Q^2)$	-3.41 ± 0.45	0.42 ± 0.03	4.59 ± 0.48
$F_{2,1}(Q^2)$	1.30 ± 0.15	0.38 ± 0.01	3.27 ± 0.40
$F_{4,0}(Q^2)$	1.38 ± 0.19	0.51 ± 0.04	3.32 ± 0.04
$F_{4,1}(Q^2)$	-0.35 ± 0.03	0.14 ± 0.01	5.41 ± 0.16
$F_{5,0}(Q^2)$	-0.13 ± 0.02	1.14 ± 0.07	2.17 ± 0.01

Ref. [55] result. When the $\mathcal{D}_2^\Delta$ and $\mathcal{D}_3^\Delta$ results are examined, it is seen that our results are compatible with the results of Refs. [53,55] within the errors. The vanishing of $\mathcal{D}_2^\Delta$ in the QCD sum rule approach is significant, as it confirms the remarkable prediction of viewing baryons as the chiral solitons [53].

By means of the Δ baryon's D-terms, one can obtain the D-term of the nucleon using the large N_c picture of baryons as chiral solitons as follows [53]:

$$\mathcal{D}_0^N = \mathcal{D}_0^\Delta + 2\mathcal{D}_3^\Delta. \quad (38)$$

The above relation yields $\mathcal{D}_0^N = -3.57 \pm 0.46$ using the D-terms of the Δ baryon in our calculations, which is in good agreement with the $\mathcal{D}_0^N$ values of Refs. [53,55]. From these results, we see that the D-term $\mathcal{D}_0^N$ of nucleon has a higher absolute value than the generalized D-terms $\mathcal{D}_0^\Delta$ and $\mathcal{D}_3^\Delta$ of the Δ baryon, which are all negative as expected: It is thought that if a system satisfies the local stability conditions, the D-terms should be negative; if not the system would collapse.

V. SUMMARY AND CONCLUSION

Due to the different interaction types, a hadron can have different kinds of form factors representing the corresponding interaction. Determination of different form factors of hadrons allow us to obtain useful information about the various related physical quantities that can help us discover the nature and internal structures of hadrons as well as the nonperturbative nature of QCD as the theory of strong interaction. The gravitational form factors that emerge as a result of the gravitonlike interaction of the hadrons with the energy-momentum tensor current are of great importance as they provide important information about the inner structures, quark-gluon organizations of hadrons, distributions of the strong forces, energy, and pressure inside them as well as their geometric shape and radius. These cause an increasing interest to investigation of hadronic GFFs.

In this study, we investigated the $\Delta \to \Delta$ transition in the presence of the energy-momentum tensor current. We considered both the quark and gluonic parts of the EMT current. Such interaction is parametrized in terms of ten GFFs: seven conserved and three nonconserved form factors. The nonconserved form factors vanish because of the conservation of the total EMT current. We derived the sum rules and numerically determined the seven conserved GFFs of the Δ baryon in the range $0 \leq Q^2 \leq 10$ GeV^2 using the three-point QCD sum rules approach. The QCD sum rule method is a relativistic method and considers different features and quantum numbers of the hadrons like their spin, being one of the leading existing nonperturbaative approaches. We found that the Q^2 behavior of Δ's GFFs are well explained via a p-pole fit function. We presented the values of the GFFs at zero momentum transfer as well.

Having determined the GFFs of the Δ baryon, we used them to calculate the composite gravitational form factors of the system like the energy and angular momentum multipole form factors, $\mathcal{D}$ terms representing the mechanical properties

TABLE II. A comparison of mechanical properties obtained in the present study at zero momentum transfer with those from the Skyrme model [55].

Model	$\varepsilon_0(0)$	$\varepsilon_2(0)$	$\mathcal{J}_1(0)$	$\mathcal{J}_3(0)$	$D_0(0)$	$D_2(0)$	$D_3(0)$
This work	1.01 ± 0.15	-0.18 ± 0.03	0.46 ± 0.06	-0.17 ± 0.03	-2.71 ± 0.34	-0.17 ± 0.03	0.26 ± 0.04
[55]	1	0.34	0.5	0	−3.53	−0.20	0.24

TABLE III. A comparison of the D-terms and the mass radius obtained in the present study with those from the Skyrme model [52,53,55].

Model	$\mathcal{D}_0^\Delta$	$\mathcal{D}_2^\Delta$	$\mathcal{D}_3^\Delta$	$\mathcal{D}_0^N$	$\langle r_E^2 \rangle$(fm^2)
This work	-2.71 ± 0.34	0.000 ± 0.002	-0.43 ± 0.06	-3.57 ± 0.46	0.67 ± 0.04
[52,53]	−2.65	0	−0.38	−3.40	0.64
[55]	−3.53	0	−0.50	−3.63	0.64

like the internal pressure and shear forces, as well as the mass radius of Δ resonance and compared them with other existing theoretical predictions. Our results obtained using QCD sum rules agree with the remarkable prediction of the soliton picture of baryons, which resulted from vanishing of $\mathcal{D}_2^\Delta$ term.

Our results on $\varepsilon_0(Q^2)$, $\varepsilon_2(Q^2)$, $\mathcal{J}_1(Q^2)$, and $\mathcal{J}_3(Q^2)$, which are respectively energy-monopole, energy-quadrupole, angular momentum-dipole, and angular momentum-octupole form factors as well as $D_{0,2,3}(Q^2)$ composite form factors related to the internal pressures and shear forces and the generalized D-terms $\mathcal{D}_{0,2,3}$ satisfy the required conditions and describe well-different features of the Δ baryon. Our results may be compared with future probable lattice QCD and other theoretical predictions. We hope that such investigations will be possible in future experiments as well. If the direct measurements of the quantities considered in the present study are difficult because of the short lifetime of the Δ baryon, we hope that we can extract GPDs of this system using experimental data on different related physical quantities like electromagnetic form factors and multipole moments. As we previously mentioned, one can determine the GFFs using the extracted GPDs from the experimental data. Comparison of the obtained GFFs by this way with the results of the present study will be of great importance as was done for the nucleon in Ref. [1].

ACKNOWLEDGMENTS

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APPENDIX A: QCD SIDE SOLUTIONS OF THREE-POINT CORRELATION FUNCTION

In this appendix, we collect some parts of QCD side solutions of the three-point correlation function. In Eq. (17), after using Wick's theorem and calculating all possible contractions, Γ^q and Γ^g are obtained as below,

$$\begin{aligned}
\Gamma^q_{\alpha\mu\nu\beta} = i\Big\{ & 4S_u^{cc'}(y-x)\mathrm{Tr}\Big[\gamma_\beta S'^{bb'}_d(y-x)\gamma_\alpha S_u^{am}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{ma'}(z-x)\Big] \\
& -4S_u^{ca'}(y-x)\gamma_\beta S'^{bb'}_d(y-x)\gamma_\alpha S_u^{am}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{mc'}(z-x) \\
& -4S_u^{cm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{ma'}(z-x)\gamma_\beta S'^{bb'}_d(y-x)\gamma_\alpha S_u^{ac'}(y-x) \\
& +4S_u^{cm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{mc'}(z-x)\mathrm{Tr}\Big[\gamma_\beta S'^{bb'}_d(y-x)\gamma_\alpha S_u^{aa'}(y-x)\Big] \\
& +4S_u^{cc'}(y-x)\mathrm{Tr}\Big[\gamma_\beta S'^{aa'}_u(y-x)\gamma_\alpha S_d^{bm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_d^{mb'}(z-x)\Big] \\
& -4S_u^{ca'}(y-x)\gamma_\beta S'^{mb'}_d(z-x)\overleftrightarrow{D}_\mu(z)\gamma_\nu S'^{bm}_d(y-z)\gamma_\alpha S_u^{ac'}(y-x) \\
& +2S_u^{ca'}(y-x)\gamma_\beta S'^{mb'}_u(z-x)\overleftrightarrow{D}_\mu(z)\gamma_\nu S'^{am}_u(y-z)\gamma_\alpha S_d^{bc'}(y-x) \\
& -2S_u^{cb'}(y-x)\gamma_\beta S'^{ma'}_u(z-x)\overleftrightarrow{D}_\mu(z)\gamma_\nu S'^{am}_u(y-z)\gamma_\alpha S_d^{bc'}(y-x) \\
& +2S_u^{cm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{ma'}(z-x)\gamma_\beta S'^{ab'}_u(y-x)\gamma_\alpha S_d^{bc'}(y-x) \\
& -2S_u^{cm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{mb'}(z-x)\gamma_\beta S'^{aa'}_u(y-x)\gamma_\alpha S_d^{bc'}(y-x) \\
& +2S_u^{ca'}(y-x)\gamma_\beta S'^{ab'}_u(y-x)\gamma_\alpha S_d^{bm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_d^{mc'}(z-x) \\
& -2S_u^{cb'}(y-x)\gamma_\beta S'^{aa'}_u(y-x)\gamma_\alpha S_d^{bm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_d^{mc'}(z-x) \\
& +2S_d^{cb'}(y-x)\gamma_\beta S'^{ba'}_u(y-x)\gamma_\alpha S_u^{am}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{mc'}(z-x) \\
& -2S_d^{cb'}(y-x)\gamma_\beta S'^{ma'}_u(z-x)\overleftrightarrow{D}_\mu(z)\gamma_\nu S'^{am}_u(y-z)\gamma_\alpha S_u^{bc'}(y-x) \\
& +2S_d^{cb'}(y-x)\gamma_\beta S'^{ma'}_u(z-x)\overleftrightarrow{D}_\mu(z)\gamma_\nu S'^{bm}_u(y-z)\gamma_\alpha S_u^{ac'}(y-x) \\
& -2S_d^{cb'}(y-x)\gamma_\beta S'^{aa'}_u(y-x)\gamma_\alpha S_u^{bm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{mc'}(z-x) \\
& +2S_d^{cm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_d^{mb'}(z-x)\gamma_\beta S'^{ba'}_u(y-x)\gamma_\alpha S_u^{ac'}(y-x) \\
& -2S_d^{cm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_d^{mb'}(z-x)\gamma_\beta S'^{aa'}_u(y-x)\gamma_\alpha S_u^{bc'}(y-x)
\end{aligned}$$

$$
\begin{aligned}
&+ S_d^{cc'}(y-x)\mathrm{Tr}\Big[\gamma_\beta S'^{bb'}_u(y-x)\gamma_\alpha S_u^{am}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{ma'}(z-x)\Big]\\
&- S_d^{cc'}(y-x)\mathrm{Tr}\Big[\gamma_\beta S'^{ba'}_u(y-x)\gamma_\alpha S_u^{am}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{mb'}(z-x)\Big]\\
&- S_d^{cc'}(y-x)\mathrm{Tr}\Big[\gamma_\beta S'^{ab'}_u(y-x)\gamma_\alpha S_u^{bm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{ma'}(z-x)\Big]\\
&+ S_d^{cc'}(y-x)\mathrm{Tr}\Big[\gamma_\beta S'^{aa'}_u(y-x)\gamma_\alpha S_u^{bm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_u^{mb'}(z-x)\Big]\\
&+ S_d^{cm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_d^{mc'}(z-x)\mathrm{Tr}\Big[\gamma_\beta S'^{aa'}_u(y-x)\gamma_\alpha S_u^{bb'}(y-x)\Big]\\
&- S_d^{cm}(y-z)\gamma_\nu \overleftrightarrow{D}_\mu(z) S_d^{mc'}(z-x)\mathrm{Tr}\Big[\gamma_\beta S'^{ab'}_u(y-x)\gamma_\alpha S_u^{ba'}(y-x)\Big] + \mu \leftrightarrow \nu\Big\},
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
\Gamma^{g}_{\alpha\mu\nu\beta} = \langle G^2\rangle g_{\mu\nu}\Big\{&4S_u^{cc'}(y-x)\mathrm{Tr}\Big[\gamma_\beta S'^{bb'}_d(y-x)\gamma_\alpha S_u^{aa'}(y-x)\Big] - 4S_u^{ca'}(y-x)\gamma_\beta S'^{bb'}_d(y-x)\gamma_\alpha S_u^{ac'}(y-x)\\
&+ 2S_u^{ca'}(y-x)\gamma_\beta S'^{ab'}_u(y-x)\gamma_\alpha S_d^{bc'}(y-x) - 2S_u^{cb'}(y-x)\gamma_\beta S'^{aa'}_u(y-x)\gamma_\alpha S_d^{bc'}(y-x)\\
&+ 2S_d^{cb'}(y-x)\gamma_\beta S'^{ba'}_u(y-x)\gamma_\alpha S_u^{ac'}(y-x) - 2S_d^{cb'}(y-x)\gamma_\beta S'^{aa'}_u(y-x)\gamma_\alpha S_u^{bc'}(y-x)\\
&+ S_d^{cc'}(y-x)\mathrm{Tr}\Big[\gamma_\beta S'^{aa'}_u(y-x)\gamma_\alpha S_u^{bb'}(y-x)\Big] - S_d^{cc'}(y-x)\mathrm{Tr}\Big[\gamma_\beta S'^{ab'}_u(y-x)\gamma_\alpha S_u^{ba'}(y-x)\Big]\Big\},
\end{aligned}
\tag{A2}
$$

where $S' = CS^TC$ and $S_q^{ij}(x)$ is the light quark propagator, defined by

$$
\begin{aligned}
S_q^{ij}(x) = {}& i\delta_{ij}\frac{\not{x}}{2\pi^2x^4} - \delta_{ij}\frac{m_q}{4\pi^2x^2} - \delta_{ij}\frac{\langle\bar{q}q\rangle}{12} + i\delta_{ij}\frac{\not{x}m_q\langle\bar{q}q\rangle}{48} - \delta_{ij}\frac{x^2}{192}m_0^2\langle\bar{q}q\rangle + i\delta_{ij}\frac{x^2\not{x}m_q}{1152}m_0^2\langle\bar{q}q\rangle\\
&- i\frac{g_sG_{ij}^{\lambda\delta}}{32\pi^2x^2}[\not{x}\sigma_{\lambda\delta} + \sigma_{\lambda\delta}\not{x}] + \cdots
\end{aligned}
\tag{A3}
$$

with $m_0^2 = \langle\bar{q}g_sG^{\mu\nu}\sigma_{\mu\nu}q\rangle/\langle\bar{q}q\rangle$ and we assume $m_q = 0$.

The perturbative and nonperturbative contributions of the correlation function in Eq. (19) are given by

$$
\begin{aligned}
\Gamma^{(P)}_{\alpha\mu\nu\beta} = \frac{3i^7}{(2\pi^2)^4}\frac{1}{(y-x)^8}\Big\{&2(\not{y}-\not{x})\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^P_{\mu\nu}(x,y)\Big] + 2(\not{y}-\not{x})\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^P_{\mu\nu}(x,y)\\
&+ 2A^P_{\mu\nu}(x,y)\gamma_\beta(\not{y}-\not{x})\gamma_\alpha(\not{y}-\not{x}) + A^P_{\mu\nu}(x,y)\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha(\not{y}-\not{x})\Big] + 2(\not{y}-\not{x})\gamma_\beta B^P_{\mu\nu}(x,y)\gamma_\alpha(\not{y}-\not{x})\Big\},
\end{aligned}
\tag{A4}
$$

$$
\begin{aligned}
\Gamma^{(3D)}_{\alpha\mu\nu\beta} = {}& \frac{i^6}{4(2\pi^2)^3}\frac{\langle\bar{q}q\rangle}{(y-x)^4}\Big\{2(\not{y}-\not{x})\mathrm{Tr}\Big[\gamma_\beta\gamma_\alpha A^P_{\mu\nu}(x,y)\Big] - 2\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^P_{\mu\nu}(x,y)\Big] + 2(\not{y}-\not{x})\gamma_\beta\gamma_\alpha A^P_{\mu\nu}(x,y)\\
&- 2\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^P_{\mu\nu}(x,y) + 2A^P_{\mu\nu}(x,y)\gamma_\beta\gamma_\alpha(\not{y}-\not{x}) - 2A^P_{\mu\nu}(x,y)\gamma_\beta(\not{y}-\not{x})\gamma_\alpha + A^P_{\mu\nu}(x,y)\mathrm{Tr}\Big[\gamma_\beta\gamma_\alpha(\not{y}-\not{x})\Big]\\
&- A^P_{\mu\nu}(x,y)\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha\Big] - 2(\not{y}-\not{x})\gamma_\beta B^P_{\mu\nu}(x,y)\gamma_\alpha - 2\gamma_\beta B^P_{\mu\nu}(x,y)\gamma_\alpha(\not{y}-\not{x})\\
&- \frac{1}{(y-x)^4}(2(\not{y}-\not{x})\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^3_{\mu\nu}(x,y)\Big] + 2(\not{y}-\not{x})\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^3_{\mu\nu}(x,y)\\
&+ 2A^3_{\mu\nu}(x,y)\gamma_\beta(\not{y}-\not{x})\gamma_\alpha(\not{y}-\not{x}) + A^3_{\mu\nu}(x,y)\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha(\not{y}-\not{x})\Big] + 2(\not{y}-\not{x})\gamma_\beta B^3_{\mu\nu}(x,y)\gamma_\alpha(\not{y}-\not{x}))\Big\},
\end{aligned}
\tag{A5}
$$

$$
\begin{aligned}
\Gamma^{(4D,q)}_{\alpha\mu\nu\beta} = \frac{i^7}{24(8\pi^2)^4}\frac{g_s^2\langle G^2\rangle}{(y-x)^4}\Big\{&2((\not{y}-\not{x})\sigma^{\lambda\delta} + \sigma^{\lambda\delta}(\not{y}-\not{x}))\mathrm{Tr}\Big[\gamma_\beta\Big((\not{y}-\not{x})\sigma_{\lambda\delta} + \sigma_{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\alpha A^P_{\mu\nu}(x,y)\Big]\\
&+ 2\Big((\not{y}-\not{x})\sigma^{\lambda\delta} + \sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\beta\Big((\not{y}-\not{x})\sigma_{\lambda\delta} + \sigma_{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\alpha A^P_{\mu\nu}(x,y)\\
&+ 2A^P_{\mu\nu}(x,y)\gamma_\beta\Big((\not{y}-\not{x})\sigma^{\lambda\delta} + \sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\alpha\Big((\not{y}-\not{x})\sigma_{\lambda\delta} + \sigma_{\lambda\delta}(\not{y}-\not{x})\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+A^{P}_{\mu\nu}(x,y)\mathrm{Tr}\Big[\gamma_\beta\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\alpha\Big((\not{y}-\not{x})\sigma_{\lambda\delta}+\sigma_{\lambda\delta}(\not{y}-\not{x})\Big)\Big]\\
&-2\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\beta B^{P}_{\mu\nu}(x,y)\gamma_\alpha\Big((\not{y}-\not{x})\sigma_{\lambda\delta}+\sigma_{\lambda\delta}(\not{y}-\not{x})\Big)\\
&+\frac{1}{2(y-x)^4}(2(\not{y}-\not{x})\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^{G^2}_{\mu\nu}(x,y)\Big]+2(\not{y}-\not{x})\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^{G^2}_{\mu\nu}(x,y)\\
&+2A^{G^2}_{\mu\nu}(x,y)\gamma_\beta(\not{y}-\not{x})\gamma_\alpha(\not{y}-\not{x})+A^{G^2}_{\mu\nu}(x,y)\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha(\not{y}-\not{x})\Big]+2(\not{y}-\not{x})\gamma_\beta B^{G^2}_{\mu\nu}(x,y)\gamma_\alpha(\not{y}-\not{x}))\\
&+\frac{1}{(y-x)^2}(2(\not{y}-\not{x})\mathrm{Tr}\Big[\gamma_\beta((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x}))\gamma_\alpha A^{PG}_{\mu\nu}(x,y)\Big]\\
&-2\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^{PG}_{\mu\nu}(x,y)\Big]\\
&+2(\not{y}-\not{x})\gamma_\beta\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\alpha A^{PG}_{\mu\nu}(x,y)-2\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^{PG}_{\mu\nu}(x,y)\\
&+2A^{PG}_{\mu\nu}(x,y)\gamma_\beta\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\alpha(\not{y}-\not{x})-2A^{PG}_{\mu\nu}(x,y)\gamma_\beta(\not{y}-\not{x})\gamma_\alpha\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\\
&+A^{PG}_{\mu\nu}(x,y)\Big(\mathrm{Tr}\Big[\gamma_\beta\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\alpha(\not{y}-\not{x})\Big]-\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\Big]\Big)\\
&+2(\not{y}-\not{x})\gamma_\beta B^{PG}_{\mu\nu}(x,y)\gamma_\alpha\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)+2\Big((\not{y}-\not{x})\sigma^{\lambda\delta}+\sigma^{\lambda\delta}(\not{y}-\not{x})\Big)\gamma_\beta B^{PG}_{\mu\nu}(x,y)\gamma_\alpha(\not{y}-\not{x})\Big\},
\end{aligned}
\tag{A6}
$$

$$
\Gamma^{(4D,g)}_{\alpha\mu\nu\beta}=\frac{6i^5}{(2\pi^2)^3}\frac{\langle G^2\rangle g_{\mu\nu}}{(y-x)^{12}}\Big\{(\not{y}-\not{x})\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha(\not{y}-\not{x})\Big]+2(\not{y}-\not{x})\gamma_\beta(\not{y}-\not{x})\gamma_\alpha(\not{y}-\not{x})\Big\},
\tag{A7}
$$

$$
\begin{aligned}
\Gamma^{(5D)}_{\alpha\mu\nu\beta}=\frac{i^6}{(8\pi^2)^3}\frac{m_0^2\langle\bar{q}q\rangle}{(y-x)^2}\Big\{&2(\not{y}-\not{x})\mathrm{Tr}\Big[\gamma_\beta\gamma_\alpha A^{P}_{\mu\nu}(x,y)\Big]-2\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^{P}_{\mu\nu}(x,y)\Big]+2(\not{y}-\not{x})\gamma_\beta\gamma_\alpha A^{P}_{\mu\nu}(x,y)\\
&-2\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^{P}_{\mu\nu}(x,y)+2A^{P}_{\mu\nu}(x,y)\gamma_\beta\gamma_\alpha(\not{y}-\not{x})-2A^{P}_{\mu\nu}(x,y)\gamma_\beta(\not{y}-\not{x})\gamma_\alpha+A^{P}_{\mu\nu}(x,y)\mathrm{Tr}\Big[\gamma_\beta\gamma_\alpha(\not{y}-\not{x})\Big]\\
&-A^{P}_{\mu\nu}(x,y)\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha\Big]-2(\not{y}-\not{x})\gamma_\beta B^{P}_{\mu\nu}(x,y)\gamma_\alpha-2\gamma_\beta B^{P}_{\mu\nu}(x,y)\gamma_\alpha(\not{y}-\not{x})\\
&+\frac{1}{(y-x)^6}\Big(2(\not{y}-\not{x})\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^{5}_{\mu\nu}(x,y)\Big]+2(\not{y}-\not{x})\gamma_\beta(\not{y}-\not{x})\gamma_\alpha A^{5}_{\mu\nu}(x,y)\\
&+2A^{5}_{\mu\nu}(x,y)\gamma_\beta(\not{y}-\not{x})\gamma_\alpha(\not{y}-\not{x})+A^{5}_{\mu\nu}(x,y)\mathrm{Tr}\Big[\gamma_\beta(\not{y}-\not{x})\gamma_\alpha(\not{y}-\not{x})\Big]+2(\not{y}-\not{x})\gamma_\beta B^{5}_{\mu\nu}(x,y)\gamma_\alpha(\not{y}-\not{x})\Big)\Big\},
\end{aligned}
\tag{A8}
$$

where,

$$
\begin{aligned}
A^{P}_{\mu\nu}(x,y)&=\frac{y}{y^4}\gamma_\nu\left[\frac{\gamma_\mu}{x^4}-\frac{4\not{x}x_\mu}{x^6}\right]-\left[\frac{\gamma_\mu}{y^4}-\frac{4\not{y}y_\mu}{y^6}\right]\gamma_\nu\frac{\not{x}}{x^4},\\
A^{3}_{\mu\nu}(x,y)&=\gamma_\nu\left[\frac{\gamma_\mu}{x^4}-\frac{4\not{x}x_\mu}{x^6}\right]+\left[\frac{\gamma_\mu}{y^4}-\frac{4\not{y}y_\mu}{y^6}\right]\gamma_\nu,\\
A^{G^2}_{\mu\nu}(x,y)&=\left[\frac{\not{y}\sigma^{\lambda\delta}+\sigma^{\lambda\delta}\not{y}}{y^2}\right]\gamma_\nu\left[\left(\frac{\gamma_\mu}{x^2}-\frac{2\not{x}x_\mu}{x^4}\right)\sigma_{\lambda\delta}+\sigma_{\lambda\delta}\left(\frac{\gamma_\mu}{x^2}-\frac{2\not{x}x_\mu}{x^4}\right)\right]-\left[\left(\frac{\gamma_\mu}{y^2}-\frac{2\not{y}y_\mu}{y^4}\right)\sigma^{\lambda\delta}+\sigma^{\lambda\delta}\left(\frac{\gamma_\mu}{y^2}-\frac{2\not{y}y_\mu}{y^4}\right)\right]\gamma_\nu\left[\frac{\not{x}\sigma_{\lambda\delta}+\sigma_{\lambda\delta}\not{x}}{x^2}\right],\\
A^{PG}_{\mu\nu}(x,y)&=\frac{\not{y}}{y^4}\gamma_\nu\left[\left(\frac{\gamma_\mu}{x^2}-\frac{2\not{x}x_\mu}{x^4}\right)\sigma_{\lambda\delta}+\sigma_{\lambda\delta}\left(\frac{\gamma_\mu}{x^2}-\frac{2\not{x}x_\mu}{x^4}\right)\right]-\left[\frac{\gamma_\mu}{y^4}-\frac{4\not{y}y_\mu}{y^6}\right]\gamma_\nu\left[\frac{\not{x}\sigma_{\lambda\delta}+\sigma_{\lambda\delta}\not{x}}{x^2}\right]\\
&\quad+\left[\frac{\not{y}\sigma_{\lambda\delta}+\sigma_{\lambda\delta}\not{y}}{y^2}\right]\gamma_\nu\left[\frac{\gamma_\mu}{x^4}-\frac{4\not{x}x_\mu}{x^6}\right]-\left[\left(\frac{\gamma_\mu}{y^2}-\frac{2\not{y}y_\mu}{y^4}\right)\sigma_{\lambda\delta}+\sigma_{\lambda\delta}\left(\frac{\gamma_\mu}{y^2}-\frac{2\not{y}y_\mu}{y^4}\right)\right]\gamma_\nu\frac{\not{x}}{x^4},\\
A^{5}_{\mu\nu}(x,y)&=\frac{2\gamma_\nu\not{x}y_\mu}{x^4}-y^2\gamma_\nu\left[\frac{\gamma_\mu}{x^4}-\frac{4\not{x}x_\mu}{x^6}\right]-\left[\frac{\gamma_\mu}{y^4}-\frac{4\not{y}y_\mu}{y^6}\right]\gamma_\nu x^2+\frac{2\not{y}\gamma_\nu x_\mu}{y^4},\\
B_{\mu\nu}(x,y)&=A_{\mu\nu}(y,x).
\end{aligned}
\tag{A9}
$$

APPENDIX B: GLUON CONDENSATION

The light quark propagator in Eq. (A3) includes one gluon strength field tensor $-ig_s G_{ij}^{\lambda\delta}[\not{x}\sigma_{\lambda\delta} + \sigma_{\lambda\delta}\not{x}]/32\pi^2 x^2$. A two-gluon condensation can be formed by multiplying these terms together in the presence of vacuum. We simplify such expressions with these notations [72],

$$G_{ab}^{\alpha\beta} = G_A^{\alpha\beta} t_{ab}^A, \qquad t^A = \frac{1}{2}\lambda^A, \qquad G^2 = G_{\alpha\beta}^A G_{\alpha\beta}^A,$$
$$t_{ab}^A t_{a'b'}^A = \frac{1}{2}\left(\delta_{ab'}\delta_{a'b} - \frac{1}{3}\delta_{ab}\delta_{a'b'}\right), \tag{B1}$$

where $a, b = 1, 2, 3$ and $A = 1, 2, \ldots, 8$ are color indices of the fundamental (quark) and the adjoint (gluon) representations, respectively and λ^A are Gell-Mann matrices. We consider $\langle 0|G_{\alpha\beta}^A(x) G_{\alpha'\beta'}^{A'}(0)|0\rangle$ as the gluon condensate and use the first term of the Taylor expansion at $x = 0$,

$$\langle 0|G_{\alpha\beta}^A(0) G_{\alpha'\beta'}^{A'}(0)|0\rangle = \frac{\langle G^2\rangle}{96}\delta^{AA'}\left[g_{\alpha\alpha'}g_{\beta\beta'} - g_{\alpha\beta'}g_{\alpha'\beta}\right]. \tag{B2}$$

We apply Eqs. (B1) and (B2) to Eqs. (A2) and (A6).

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