A NOTE ON WEAKLY PRIME HYPERIDEALS AND $(1, n)$ HYPERIDEALS ON MULTIPLICATIVE HYPERRINGS

by

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In this article we give the definition of weakly prime hyperideals over multiplicative hyperring. We provide important results showing the relations between prime hyperideals and weakly prime hyperideals. Then we give definition of weakly n-hyperideal and weakly (1, n) hyperideal over multiplicative hyperrings. Then we investigate of their properties and give some examples. Hyperstructures are very important topic because it satisfy to study multidisiplinary. Especially in chemistry, coding theory, geometry, artificial intelligence, etc.

Key words: *multiplicative hyperrings, n-hyperideal, (1, n) hyperideal, weakly hyperideal*

Introduction

Marty is the pioneer of the hyperstructure theory, in 1934 [1]. Hyperstructures have many utilization either pure mathematics or applied mathematics. A hypergroup is a non-empty set *H* endowed by a hyperoperation $*: H \times H \to P^*(H)$, where *P*(*H*) is the set of all non-empty subsets of *H*, which satisfies the associative law and reproduction axiom. Canonical hypergroups is a special class of the hypergroup of Marty. The more general structure that satisfies the ring-like axioms is the hyperring in the general sense: (*R*, +) is a hyperring if "+" and "." are two hyperopertions such that (*R*, +) is a hypergroup and "." is an associative hyperoperation, which is distributive with respect to "+". There are different notions of hyperrings [1]. If only the addition "+" is hyperoperation and the multiplication "." is usual operation, then we say that *R* is an additive hyperring. A special case of this type is the hyperring introduced by Krasner [2]. Also, Krasner in 1983 introduced a class of hyperring and and hyperfields, the quotient hyperrings and hyperfields [2]. If only "." is a hyperoperation, we shall say that *R* is a multiplicative hyperring. Rota (1982) introduced the multiplicative hyperrings; subsequently, many authors work on this field (Nakassis, 1988; Olson and Ward, 1997; Procesi and Rota,1999; Rota, 1996) [2]. Algebraic hyperstructures has been studied in the following decades and nowadays by many mathematicians. Weakly prime ideals was introduced by Anderson and Smith(2003). They define weakly prime ideals and gave some important properties of weakly prime ideals [3].

Let H be a non-empty set and $\circ: H \times H \to P^*(H)$ be a hyperoperation. The couple $(H, \circ)$ is called hypergroupoid. For any two non-empty subsets *A* and *B* of *H*, we define $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$. A hypergroupoid $(H, \circ)$ is called a semihypergroup if for all *a, b, c* of *H*

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we have $a\circ(b\circ c)=(a\circ b)\circ c$, which means that $\bigcup_{u\in a\circ b} u\circ c=\bigcup_{v\in b\circ c} a\circ v$. A hypergrupoid $(H,\circ)$ is called quasihypergroup if for all a of H we have $a\circ H=H\circ a=H.$ A hypergrupoid $(H,\circ)$ which is both a semihypergroup and a quasihypergroup is called a hypergroup [2].

A triple $(R,+,.)$ is called a multiplicative hyperring if:

1) $(R, +)$ is an abelian group
2) $(R, .)$ is a semihypergroup
3) For all $a,b,c\in R$, we have $a.(b+c)\subseteq a.b+a.c$ and $(b+c).a\subseteq b.a+c.a$
4) For all $a,b\in R$, we have $a.(-b)=(-a).b=-(a.b)$ [2].

If in (3) we have equalities instead of inclusions, then we say that the multiplicative hyperring is strongly distributive. A non-empty subset I of a commutative hyperring R is said to be a hyperideal of R if $x-y\in I$ and $r.x\subseteq I$ for any $x,y\in I$ and r is element of R. If A and B are hyperideals of R, then $A+B=\{a+b : a\in A, b\in B\}$ and $A.B=\bigcup\{\Sigma_{i=1}^{n} a_i.b_i : a_i\in A, b_i\in B, n\in\mathbb{Z}\}$ which are hyperideals of R. The principal ideal of R generated by an element a is given by:

$$\langle a\rangle=\{p.a : p\in\mathbb{Z}\}+\{\sum_{i=1}^{n}x_i+\sum_{j=1}^{m}y_i+\sum_{k=1}^{z}z_k : \forall i,j,k,\exists r_i,s_j,$$
$$u_k\in R \quad \text{such that} \quad x_i\in r_i\circ a, y_j\in a\circ s_j, z_k\in t_k\circ a\circ u_k\}$$

We define a set $A=\{r\in R : r^n\subseteq I\}$ for some $n\in\mathbb{N}$ is called the radical of I and denoted by $\sqrt{I}$ or $rad(I)$. A homomorphism (good homomorphism) between two multiplicative hyperrings $(R,+,\circ)$ and $(R',+',\circ')$ is a map $f:R\to R'$ such that for all $x,y\in R,$ we have $f(x+y)=f(x)+'f(y)$ and $f(x\circ y)\subseteq f(x)\circ' f(y)$ $[f(x\circ y)=f(x)\circ' f(y)]$ respectively [3]. Let R be a commutative ring with identity. We define a proper hyperideal P of R to be weakly prime if $0\neq a.b\in P$ implies $a\in P$ or $b\in P.$ For any hyperideals I and J of R, $(I:J)=\{a\in R : a\circ J\subseteq I\}.$ In [4] authors give the definition of n-ideal. They gave some properties of n-ideals. In [5] author gave the definiton r-ideal. An element $a\in R$ is called a regular (resp., zerodivisor) element if $Ann(a)=(0_R),$ $Ann(a)\neq(0_R)$ and $Ann(a)=\{r\in R : a.r=0_R\}.$ We will show the set of all regular elements and zero divisor elements of R, $r(R)$ and $zd(R)$, esp. Then he defined r-ideal (esp., pr-ideal) of R if whenever $a.b\in I$ and $a\in r(R)$ imply that $b\in I$ ($b^n\in I$ for some natural number n). Ameri and Norouzi [1] gave the definition of a nilpotent element. An element $a\in R$ is called nilpotent, if $0_R=a^n=a\circ a\circ a\circ....\circ a,$ for some $n\in\mathbb{N}.$ Badawi and Celikel [6] gived the definiton of 1-absorbing primary ideal. If $a.b.c\in I$ for some non-units $a,b,c\in A$ implies that either $a.b\in I$ or $c\in\sqrt{I}.$ It is the generalization of primary ideals. Yassine *et al.* [7] defined 1-absorbing prime ideal if $a.b.c\in I$ for some non-unit $a,b,c\in A$ implies that $a.b\in I$ or $c\in I.$ In [8] authors define weakly 1-absorbing prime ideals and give some properties of them. Let A be a ring and a proper ideal P of A is said to be a weakly 1-absorbing ideal if for each non-unit $x,y,z\in I$ with $0\neq xyz\in P$ then either $xy\in P$ or $z\in P.$ Ugurlu [9] define a new concept of n-hyperideals of commutative multiplicative hyperring. She gaved some important properties and characterized n-hyperideals. In [10] authors defined strongly 1-absorbing primary ideals if whenever $abc\in I$ for some non-units $a,b,c\in I,$ then $ab\in I$ or $c\in\eta(A).$ Here $\eta(A)$ is the set of all nilpotent elements of A. In this study we investigate weakly prime hyperideal over commutative multiplicative hyperrings and their important properties. Then we give weakly n-hyperideal and weakly (1, n) hyperideal and some important properties about them.

Weakly prime hyperideals

Throughout this paper *R* denotes the multiplicative hyperring.

Definition 1. Let *R* be a multiplicative hyperring and *P* be a proper hyperideal of *R*. If $\{0\} \neq a \circ b \subseteq P$ implies $a \in P$ or $b \in P$ then we say *P* is a weakly prime hyperideal.

Example 1. By the definitions of weakly prime hyperideal and prime hyperideal, every prime hyperideal is a weakly prime hyperideal. But the converse is not true. See the example below.

Example 2. See the example below from [1]. Let take $(\mathbb{Z}_4, \oplus, \otimes)$ on the set of integer.

Table 1. The list of addition of elements on $\mathbb{Z}_4$

$\oplus$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{0}$
$\bar{2}$	$\bar{2}$	$\bar{3}$	$\bar{0}$	$\bar{1}$
$\bar{3}$	$\bar{3}$	$\bar{0}$	$\bar{1}$	$\bar{2}$

Table 2. The list of multiplication of elements on $\mathbb{Z}_4$

$\oplus$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$
$\bar{1}$	$\bar{0}$	$\mathbb{Z}_4$	$\{\bar{0},\bar{2},\}$	$\mathbb{Z}_4$
$\bar{2}$	$\bar{0}$	$\{\bar{0},\bar{2},\}$	$\bar{0}$	$\{\bar{0},\bar{2},\}$
$\bar{3}$	$\bar{0}$	$\mathbb{Z}_4$	$\{\bar{0},\bar{2},\}$	$\mathbb{Z}_4$

Description: From tabs. 1 and 2 $(\mathbb{Z}_4, \oplus, \otimes)$ is a multiplicative commutative hyperring on the set of integers. From the tab. 2 the hyperideal $\{\bar{0}, \bar{2}\}$ is a prime hyperideal of $(\mathbb{Z}_4, \oplus, \otimes)$. From the definition we can say that it is weakly prime hyperideal. But $\bar{0}$ is not prime hyperideal and it is weakly prime hyperideal of $\mathbb{Z}_4$.

Example 3. Let define the example in [11], take $A = \{14, 21\}$ and then $x \circ y = \{14xy, 21xy\}$ on $\mathbb{Z}$.

$\{0\} \neq 1 \circ 1 = \{14, 21\} \subseteq 7\mathbb{Z}$ but $1 \notin 7\mathbb{Z}$. Then we can say $7\mathbb{Z}$ is not weakly prime hyperideal.

Example 4. The hyperideals of $7\mathbb{Z}$, $\{0\}$ and itself because it is a hyperfield. From the definition of weakly prime hyperideal itself can not be a weakly prime hyperideal. But $\{0\}$ is a weakly prime hyperideal.

Proposition 1. Let *R* be a hyperring and *I* be a hyperideal of *R*. For $a, b \in R$, let define $a * b = a \circ b + I$. Then $(R, +, *)$ is a multiplicative hyperring. Think, a weakly prime hyperideal *P* of *R*. We will show the new set is a weakly prime hyperideal or not?

Proof. Let $\{0\} \neq a * b \subseteq P$. Then, $\{0\} \neq a * b = a \circ b + I = (a + I) \circ (b + I) \subseteq P$ and *P* is a weakly prime hyperideal then $a + I \in P$ or $b + I \in P$. By this way we get $a \in P$ or $b \in P$.

Example 5. $\{0\}$ hyperideal is not a prime hyperideal. And $(\{0\})^2 = \{0\}$.

$\{0\}$ hyperideal is not a prime hyperideal and $(\{0\})^2 = \{0\}$. It is understand that this statement is generally true in the proposition we have given below.

Theorem 1. Let *R* be a multiplicative hyperring and *P* be a weakly prime hyperideal of *R*. If *P* is not a prime hyperideal then $P^2 = \{0\}$.

Proof. Suppose that $P^2 \neq \{0\}$. We will show *P* is prime or not ? Let $a \circ b \subseteq P$. If $\{0\} \neq a \circ b \subseteq P$ then $a \in P$ or $b \in P$. Lets investigate the situation $a \circ b = \{0\}$. For a $p_0 \in P$ suppose $a \circ p_0 \neq \{0\}$. Then, $\{0\} \neq a \circ p_0 = a \circ (b + p_0) \subseteq P$. By this way, $a \in P$ or $b + p_0 \in P$. Then, $a \in P$ or $b \in P$. Let $a \circ P = 0$. For $p_0 \in P$ there is a contradiction about the hold $\{0\} = a \circ p_0$. Since $P^2 \neq \{0\}$ there is $p_0, q_0 \in P$ such that $p_0 \circ q_0 \neq \{0\}$.

Corollary 1. Let *P* be a weakly prime hyperideal of *R*. If $P \subset \sqrt{0}$ then *P* is not a prime hyperideal. If $\sqrt{0} \subset P$ then *P* is a prime hyperideal.

Proof. Suppose that $\{0\} \neq a \circ b \subseteq P$. Then we can say $a \in P$ or $b \in P$. Take, $\sqrt{0} \subset P$. Then, $\{0\} \subset P^2$ and $\{0\} \subset p_0 \circ q_0 = (a + p_0) \circ (b + q_0) \subseteq P$. Since *P* is weakly prime hyperideal $a + p_0 \in P$ or $b + q_0 \in P$. If $a \in P$ then $p_0 \in P$ else $b \in P$ then $q_0 \in P$. By this way we get *P* is a prime hyperideal. Suppose that $P \subset \sqrt{0}$. $\{0\} \neq a \circ b \subset \sqrt{0}$ and from definition *P* is not a prime hyperideal.

Theorem 2. Let *R* be a multiplicative hyperring and *P* be a proper hyperring of *R*. Then the followings are equivalent:

i. *P* is a weakly prime hyperideal
ii. For $x \in R - P$, $(P : x) = P \cup (0 : x)$
iii. For $x \in R - P$, $(P : x) = P$ or $(P : x) = (0 : x)$
iv. If $\{0\} \neq A \circ B \subseteq P$ then either $A \subseteq P$ or $B \subseteq P$

Proof. (i) $\Rightarrow$ (ii): Let $a \in (P : x)$. From the definition $a \circ x \subseteq P$. If $a \circ x \neq \{0\}$ and since *P* is a weakly prime hyperideal then $a \in P$. If $a \circ x = \{0\}$ then $a \circ x \subseteq \{0\}$ and $a \in (\{0\} : x)$. In brief we get, $a \in P \cup (0 : x)$. Then; $(P : x) \subseteq P \cup (0 : x)$ .....(1)

Let $b \in P \cup (0 : x)$. By this way we get $b \in P$ or $b \in (0 : x)$. If, $b \in P$ then, $\forall x \in R, b \circ x \subseteq P$. Then we get $b \in (P : x)$. In a short word, $(P : x) \subseteq P \cup (0 : x)$ .....(2)

If $b \in (0 : x), b \circ x \subseteq \{0\}$. Then, $b \circ x \subseteq P$ and $b \in (P : x)$.

(ii) $\Rightarrow$ (iii): obvious.

(iii) $\Rightarrow$ (i): Let take $(P : x) = P$ and $(P : x) \neq (0 : x)$. Then for all $a \in (P : x)$ and $a \notin (0 : x), a \circ x \subseteq P$. From assumption $x \notin P$ and from $(P : x) = P$ we get $a \in P$. In addition to this, from $(P : x) \neq (0 : x)$ we get $\{0\} \neq a \circ x$. By this way we say that *P* is a weakly prime hyperideal of *R*.

(iv) $\Rightarrow$ (i): Take $\{0\} \neq a \circ b \subseteq P$. $\{0\} \neq (a) \circ (b)$ then $(a) \subseteq P$ or $(b) \subseteq P$. In other word, $a \in P$ or $b \in P$.

Corollary 2. Let *R* be a multiplicative hyperring and *P*, *Q* are weakly prime hyperideals of *R* but they are not prime hyperideals. Then $P \circ Q = \{0\}$.

Proof. From the *Corollary 1* and *Theorem 2* $P \subseteq \sqrt{0}$ and $Q \subseteq \sqrt{0}$ then $P \circ Q \subseteq P \circ \sqrt{0} = \{0\}$. But the other situation is not true. In other word $P \circ \sqrt{0} = \{0\}$ then *P* is a weakly prime hyperideal and it is not a prime hyperideal. See the example below.

Example 6. Consider the set of integers; $(\mathbb{Z}, +, .)$. Define the multiplication $x \circ y = \{2xy, 3xy\}$. Then $(\mathbb{Z}, +, \circ)$ is a multiplicative hyperring. Take $P = 12\mathbb{Z}$. $\sqrt{0} = \{a \in \mathbb{Z} : a^n \subseteq 0\} = \{0\}$ and $P \circ \sqrt{0} = 12\mathbb{Z} \circ \sqrt{0} = \{0, 0, 0, ..., 0\}.$, *P* is not a prime hyperideal. Take $a = 2$ and $b = 6$ then $\{0\} \neq a \circ b = \{24, 36\} \subseteq P = 12\mathbb{Z}$ but $a \notin P$ and $b \notin P$ and *P* is not weakly prime hyperideal.

Weakly n-hyperideal and weakly (1, *n*) hyperideal

Definition 2.

i. Let I be a proper hyperideal of a hyperring R. Then, I is said to be a weakly n-hyperideal if whenever $\{0\} \neq a \circ b \subseteq I$ for some $a,b \in I,$ then $a \in I$ or $b \in \eta(R)$.
ii. I is said to be weakly (1, *n*) hyperideal if whenever $\{0\} \neq a \circ b \circ c \subseteq I$ for some non-units $a,b,c \in R,$ then $a \circ b \subseteq I$ or $c \in \eta(R)$.

Because of the definitions of weakly (1, *n*) hyperideal and weakly *n*-hyperideal, we can not say a weakly (1, *n*) hyperideal is also a *n*-hyperideal or a weakly *n*-hyperideal is also a weakly (1, *n*) hyperideal. But in abstract algebra this situation is different. Every weakly *n*-ideal is also a weakly (1, *n*) ideal. In hyperstructures this relation isn't satisfied.

Example 7. Let take $A = \{2,3\}$ on the set of integer. Think the $6\mathbb{Z}$ hyperideal of $\mathbb{Z}$. Take $a = 3, b = 6;$ $a \circ b = \{36,72\} \subseteq 6\mathbb{Z}$ neither $3 \in 6\mathbb{Z}$ nor $6 \in \eta(\mathbb{Z})$. Then we see that $6\mathbb{Z}$ isn't a weakly *n*-hyperideal.

Example 8. Let take $A = \{2,3\}$ on the set of integer. Think the $7\mathbb{Z}$ hyperideal of $\mathbb{Z}$. Take $a = 2, b = 3, c = 7.$ $a \circ b \circ c = \{168,252,378\} \subseteq 7\mathbb{Z}$ but neither $2 \circ 3 \subseteq 7\mathbb{Z}$ nor $7 \in \eta(\mathbb{Z})$, then $7\mathbb{Z}$ isn't a weakly (1, *n*) hyperideal of $\mathbb{Z}$.

Definition 3. Let R be a hyperring and I be a hyperideal of R. Then I is said to be weakly 1-absorbing primary hyperideal if for some non-units $a,b,c \in R$ and $\{0\} \neq a \circ b \circ c \subseteq I$ then $a \circ b \subseteq I$ or $c \in \sqrt{I}$.

Proposition 2. Every weakly (1, *n*) hyperideal of R is weakly 1-absorbing primary hyperideal of R.

Proof. Assume that I is a weakly (1, *n*) hyperideal of R. Suppose $\{0\} \neq a \circ b \circ c \subseteq I$ and $c \notin \sqrt{I}$, for some non-units $a,b,c \in I$ then $c \notin \eta(R)$. Since I is a weakly (1, *n*) hyperideal of R, we get $a \circ b \subseteq I$.

Proposition 3. Let R be a hyperring. Every *n*-hyperideal of R is also a weakly *n*-hyperideal of R.

Proof. It is obvious from the definition. But the converse is may not be true in every time. See the example below.

Example 9. See the *Example 2.* $\{0\}$ is a weakly *n*-hyperideal but it isn't a *n*-hyperideal.

Proposition 4. Every weakly hyperideal of a hyperring R is also a *n*-hyperideal of R.

Proof. This situation is obvious from the definition of weakly hyperideal and weakly *n*-hyperideal.

Proposition 5. Let $\{I_i\}_{i\in\Delta}$ be a non- empty set of weakly *n*-hyperideals of R. Then $\bigcap I_i$ is a weakly *n*-hyperideal of R.

Proof. Suppose that $\{0\} \neq a \circ b \subseteq \bigcap I_i$ and $a \notin \eta(R)$ for $a,b \in R.$ Then we get $\{0\} \neq a \circ b \subseteq I_i$. Because of I_i is a weakly *n*-hyperideal of R, $b \in I_i$ for $\forall i$. By this way we get $b \in \bigcap I_i$. That is, $\bigcap I_i$ is a weakly *n*-hyperideal of R.

Theorem 3. Let R be a hyperring and I be a proper hyperideal of R. Then the followings are equivalent.

i. I is a weakly (1, *n*) hyperideal of R.
ii. $(I : a \circ b) \subseteq (0 : a \circ b) \cup \eta(R)$ for every non-unit elements $a,b \in R$ such that $a \circ b \subsetneq I$.
iii. $(I : a \circ b) = (0 : a \circ b)$ or $(I : a \circ b) = \eta(R)$ for every non-units $a,b \in R$ such that $a \circ b \subsetneq I$.

iv. For every non-units $a,b \in R$ and proper hyperideal J of R with $\{0\} \neq a \circ b \circ J \subseteq I$ implies that $a \circ b \subseteq I$ or $J \subseteq \eta(R)$.

v. For every non-units $a \in R$ and proper hyperideals J, K of R with $\{0\} \neq a \circ J \circ K \subseteq I$ implies that $a \circ J \subseteq I$ or $K \subseteq \eta(R)$.

vi. For every proper hyperideals J, K, L and $\{0\} \neq J \circ K \circ L \subseteq I$ implies that $J \circ K \subseteq I$ or $L \subseteq \eta(R)$.

Proof. i) $\Rightarrow$ ii): Assume that I is a weakly (1, n)-hyperideal of R. Take $x \in (I : a \circ b)$ for non-units $a,b \in R$. Then we get $a \circ b \circ x \subseteq I$. If x is a unit in R, then $a \circ b \subseteq I$ and it is a contradiction with our assume. Assume that $a \circ b \circ x \neq \{0\}$ then the proof is completed. Since I is a weakly (1, n) hyperideal and $a \circ b \subsetneq I$. Then we get $x \in \eta(R)$. Hence $(I : a \circ b) \subseteq (0 : a \circ b) \cup \eta(R)$.

ii) $\Rightarrow$ iii): It is clear.

iii) $\Rightarrow$ iv): Suppose that $\{0\} \neq a \circ b \circ J \subseteq I$ but $a \circ b \subsetneq I$. If $(I : a \circ b) = (0 : a \circ b)$ then we have $J \subseteq (I : a \circ b) = (0 : a \circ b)$ and this implies that $a \circ b \circ J = \{0\}$, this is a contradiction. Thus we have $J \subseteq (I : a \circ b) \subseteq \eta(R)$ from the assumption.

iv) $\Rightarrow$ v): Suppose $\{0\} \neq a \circ J \circ K \subseteq I$ and $\{0\} \neq a \circ J \subsetneq I$ and $K \subsetneq \eta(R)$. Then there exists $j \in J$ such that $a \circ j \subsetneq I$. Since $\{0\} \neq a \circ J \circ K$, there exists $b \in J$ such that $a \circ b \circ K \subseteq I$ and I is a weakly (1, n) hyperideal of R, we get $a \circ b \subseteq I$, we can say $a \circ j \circ K = \{0\}$. If it is equal to zero we get a contradiction with previous assumption. By this way $a \circ j \circ K = \{0\}$ and $\{0\} \neq a \circ b \circ K = a \circ (j+b) \circ K \subseteq I$. Since $\{0\} \neq a \circ b \circ K = a \circ (j+b) \circ K \subseteq I$ and I is a weakly (1, n) hyperideal, $a \circ (j+b) \subseteq I$. As $a \circ (j+b) \subseteq I$ and $a \circ b \subseteq I$, $a \circ j \subseteq I$ and it is a contradiction.

v) $\Rightarrow$ vi): Assume that $\{0\} \neq J \circ K \circ L \subseteq I$, $J \circ K \subsetneq I$ and $L \subsetneq \eta(R)$. There exists $j \in J$ such that $j \circ K \subsetneq I$. If $j \circ K \circ L \neq \{0\}$ then $L \subseteq \eta(R)$ but this is a contradiction with the assumption above and we get $j \circ K \circ L = \{0\}$. Since $\{0\} \neq J \circ K \circ L$, there exists $b \in J$ such that $b \circ K \circ L \neq \{0\}$ and since I is a weakly (1, n) hyperideal $b \circ K \subseteq I$. $\{0\} \neq b \circ K \circ L = (j+b) \circ K \circ L \subseteq I$, we get $(j+b) \circ K \subseteq I$ therefore $j \circ K \subseteq I$ and this is a contradiction.

vi) $\Rightarrow$ i): Assume that $\{0\} \neq a \circ b \circ c \subseteq I$ for some non-units $a,b,c \in R$. Take $J = (a), K = (b), L = (c)$. We get, $\{0\} \neq a \circ b \circ c \subseteq (a) \circ (b) \circ (c) = J \circ K \circ L \subseteq I$. Then $a \circ b \subseteq (a \circ b) = J \circ K \subseteq I$ or $c \in (c) \subseteq \eta(R)$ it satisfies that I is a weakly (1, n) hyperideal of R.

Conclusions

In this study we give the definition of weakly prime hyperideals over multiplicative hyperring and then give some examples about weakly prime hyperideals. We provide important results showing the relations between prime hyperideal and weakly prime hyperideals. Then we give definition of weakly n-hyperideal and weakly (1, n) hyperideal over multiplicative hyperrings and some important features about them. In future, one can investigate that any other abstract algebra topic is can be implemented in hyperstructures or not.

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