Research Article

Analysis of local system behavior in the foreign exchange-market using neural networks and Monte-Carlo method for predictıon and risk assessment

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Abstract
In this study, we attempt to show the reason behind the poor estimation of the future values of foreign exchange-rate (FXR) signals under difference-equation modeling, using the neural network approach for evaluating the local system coefficients. To do this, we have splitted EUR/USD and AUD/CAD signals into many small-segments and modeled each segment as the signal representation of a linear time-invariant (LTI) system using the simple linear difference equation (LDE) formulation. After a precise segmentation of each FXR signal such that an LTI system based modeling is highly accurate in each segment, it is observed that the coefficient values of the corresponding LDEs are highly volatile, which indicates that a reliable estimation would be very difficult using LDE modeling. Although the LDE coefficients are usually observed to take values within a small range over a given FXR signal segment (sample-set), frequent sudden-jumps in coefficient values do occur, which subjects/forces the dynamics of FXR signals to undertake the dynamics of these sudden-jumps rather than the dynamics of any other deterministic or stochastic process. To support this observation, the range of variation of the LDE coefficients over each segment is analyzed to quantify the volatility of the foreign-exchange market for risk assessment.

Article highlights

- Poor estimation performance of difference-equation (DE) based FXR signal prediction is shown.
- The usual range of values that the DE coefficients take is specified.
- Poor estimation is linked to the high volatility of the coefficients over small data segments.

Keywords Foreign-exchange market · Financial time series · Prediction · Forecasting · Risk assessment · Neural networks

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1 Introduction

Accurate prediction of *Foreign-Exchange Rate* (FXR) signals has been a hot topic in computational finance for decades [1–3]. Many studies have focused on increasing the estimation accuracy for future values/segments of FXR signals [1–3, 6–10]. Consequently, many different methods or algorithms have been developed or proposed for a precise estimation. The majority of the proposed methods are a combination of deterministic and stochastic methods [4–11], while a smaller portion of them are either purely deterministic or stochastic methods [7, 12–15]. Based on our investigations, there is no reliable method that enables a precise estimation of future data segments for FXR signals. Particularly, stochastic methods have resulted in estimations of poor accuracy [11, 15]. A common observation in most of the studies in the literature, is that reliable estimations can only be made for relatively narrow time intervals [1–6, 12–15], and there is no reliable method that allows for a high estimation accuracy in the long-run [8–11]. Often, algorithms that have been proposed as reliable for long-term estimation either do not work as proposed or fail much quicker than expected due to inherent chaoticity [12–15].

The reason behind the failure of almost all proposed methods for the precise estimation of future data segments of FXR signals, is often thought as the complicated human-based social dynamics, which involve multiple sub-dynamics, each of which are complicated in themselves [16, 17]. For this reason, many studies have proposed that the FXR signals are chaotic in nature rather than stochastic [12–19]. However, stochastic analysis of FXR signals still prevails in the literature [20–27]. Nevertheless, a precise estimation of future data segments of FXR signals does not seem to be possible, as to this date there is no study that has reported a high estimation accuracy over a long time-interval.

In this study, we will attempt to show why difference-equation based modeling is not accurate for estimating future data-sets of FXR signals. For our analysis, we will decompose a given FXR signal into many segments, such that in each segment we can apply the linear time-invariant (LTI) system approximation and model the signal/segment/local system behavior via linear difference equation (LDE) formulation, or via autoregressive (AR) time-series (which are also an LDE formulation, but include a variance term for stochastic modeling). We will perform the LDE modeling over all segments and investigate the variation and volatility of the LDE coefficients, which will be evaluated using a neural-network. A small variance or low volatility in the coefficient values would indicate that the system is relatively stable and a precise estimation is easier, whereas high-variance or high-volatility of the coefficients would indicate difficulty in estimating future data samples. Hence, a reliable estimation is often possible unless the coefficients are highly volatile [28–31]. The reason for this occurence is that even if one can accurately estimate the future data samples of a given signal over a certain interval where the signal is relatively stable, high volatility would eventually cause a sharp increase/decrease in the signal value that would swallow one's profits (from prior accurate estimations) unless the sharp increase/decrease itself is accurately predicted. Therefore, if the underlying system is highly volatile, the dynamics of these sharp changes in the signal value should be the major investigation rather than a general approach that attempts perform a direct estimation based on all previous values of the signal under investigation.

Our LDE modeling based approach stems from the fact that any nonlinear time-variant system can be locally approximated as an LTI system over a small time interval. Hence, we will first perform segmentation on a given FXR signal, such that each segment is small enough to be accurately modeled via LTI system description. After that, LDE formulation will be used, and the local system behavior that corresponds to the segment under investigation will be described by the associated local-system coefficients.

Initially, we will apply the AR time-series formulation to analyze the local (LTI) system behaviour and perform various daily (24 upcoming hours or 24samples) predictions based on the available data at each segment. Throughout the paper, we will choose the segment size (or sample size) as N = {120, 240, 480, 960} samples (1 sample indicates an hour of the day) respectively. We will then gradually shift the investigated segment by 24 samples (hours) for every single shift, and perform a new daily prediction at each shift. While investigating each segment through 24-h shifts (each adjacent segments are only 24 samples apart), we will continuously compare the predicted daily-data and the actual daily-data via measuring their correlation coefficient. After measuring the correlation coefficient for each and every segment, we will examine the probability distribution function of the correlation coefficient to investigate whether the difference-equation (in this case AR time series) formulation is accurate for prediction over a small data-segment (Monte-Carlo analysis). If the distribution function of the correlation coefficient is skewed towards $\rho = 1$, then this would mean that difference-equation based modeling is accurate for prediction over a segment. For this part, we will use the built-in MATLAB functions for AR time series based prediction.

Followingly, we will formulate the local system behavior in terms of the plain LDE formulation (no bias or variance term included, hence no AR time-series description) for a pure analysis of the time variation of the coefficients. At this part, we will work on a constant segment with 960-samples (40 days), which are the last 960 samples of the FXR signal under investigation (either EUR/USD or AUD/CAD). This time, instead of using constant system coefficients to model the whole local system behaviour, we will adaptively solve for the LDE coefficients for each single sample of the segment, which means that at each sample shift, the LDE coefficients of the segment will be re-evaluated (a total of 20 coefficients will be used). This way the time variation of the LDE coefficients will be observed throughout the entire segment. Small coefficient variations over time would indicate a more stable system, whereas huge variations would indicate a highly volatile system. Here, a MATLAB-based neural-network function will be implemented for an adaptive solution of the system coefficients, which will be presented in the next section. To analyze the volatility of the system, we will plot the distribution function for various coefficients and compute their maximum attained value. Frequent recurrence of extremely high values in the adaptively-varying local system coefficients would mean that accurate estimation for future values is difficult, and would depend on understanding the dynamics behind this volatility.

In Sect. 2 (Methods), we describe the segmentation of an FXR signal data for LTI system based modeling, and how to solve for the associated LDE coefficients using AR time series modeling and/or a simple neural-network, via MATLAB based implementation. Section 3 involves the analysis of the prediction accuracy under LDE based modeling, comparison of the actual data with the predicted data, and the quantification of the accuracy via computation of the correlation coefficient of the actual and the predicted data. Section 3 also illustrates the volatility of the LDE coefficients via tables and figures. In the discussion part (Sect. 4), we discuss the significance of our findings regarding the volatility of the LDE coefficients, and their meaning within the context of earlier work on the topic.

2 Methods

2.1 Autoregressive (AR) time-series modeling (stochastic modeling)

We have initially investigated FXR signals through AR time series modeling in order to examine the estimation accuracy for difference-equation based models. For this part, we have segmented the FXR signals such that each adjacent segment is separated by 24 samples (concerning daily/24 h based estimations). Notice that if the number of training data samples (N) is large, the segments overlap greatly, though N cannot be chosen too-large to safeguard the LTI system approximation

$$\psi(n;1) = \left[s_1, s_1, \dots, s_N\right], \psi(n;2) = \left[s_{25}, s_{26}, \dots, s_{N+24}\right], \dots, \psi(n;P) = \left[s_{24P+1}, s_{24P+2}, \dots, s_{24P+N}\right] \tag{1}$$

After the segmentation of the overall FXR signal ψ, each segment is analyzed using AR time series modeling, which is again based on the LTI system approximation via LDE formulation, but with the addition of a bias term and a white noise signal

$$\psi(n,\xi) = y(n) = \sum_{k=1}^{M} a_k y(n-k) + a_0 + \sigma(n) \tag{2}$$

a_0 : *Bias term*, $\sigma(n)$: *White noise(zero mean and constant variance)*,

a_k : *LTI system coefficients*

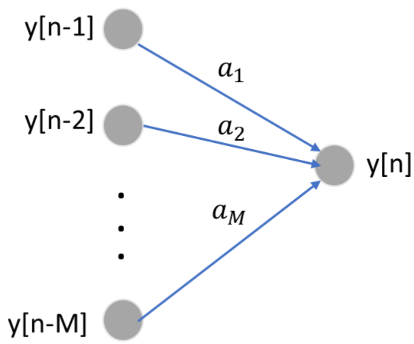
The description for the solution of all coefficients a_k is a bit involved using AR time-series modeling, and a detailed description of the process can be found in [29–32]. Here, we have used the following built-in MATLAB script to solve for the M coefficients at a given data segment ξ $(1 < \xi < P)$ of the hourly (1sample indicates an hour) EUR/USD and AUD/CAD signals (conventionally indicated as EURUSD60 and AUDCAD60 in FX Market-based computer programs). Two example runs for M = {50,20} coefficients, are given respectively for both FXR signals as follows.

m1: Beginning index of the data segment, m2: Final index of the data segment

```
>> Signal = AUDCAD60(m1:m2) >> y = iddata(Signal,[])
>> coefficients = ar(y,50)
>> Signal = EURUSD60(m1:m2) >> y = iddata(Signal,[])
>> coefficients = ar(y,50)
>> Signal = AUDCAD60(m1:m2) >> y = iddata(Signal,[])
>> coefficients = ar(y,20)
>> Signal = EURUSD60(m1:m2) >> y = iddata(Signal,[])
>> coefficients = ar(y,20)
```

2.2 Pure linear-difference-equation (LDE) based analysis over small data segments (deterministic modeling)

The FXR market signals are known to represent nonlinear time variant systems [28, 29], hence FXR signals, in their entirety, cannot be modeled using the concepts of linear time-invariant (LTI) systems. However, a relatively small segment of the overall signal can be approximated as a representation of an LTI system [7, 11], as any nonlinear time-variant system approximately behaves like an LTI system over relatively small time-intervals. Hence, a given




```
function [h,e,d,x,step] = n_network(y,h,M,iter)
step=1*10^0;
x=zeros(M,1);
delta=1*10^-6;
for k=1:iter
d=h*x;
e(k)=norm(y-d);
for i=1:M
    for j=1:1
        q=x;
        q(i,j)=q(i,j)+delta;
        dh=h*q;
        eh=norm(y-dh);
        grad=(eh-e(k));
        x(i,j)=x(i,j)-step*grad;
    end
end
end
```

Fig. 1 Neural network model of an LDE and the associated MATLAB function for the solution of its coefficients

FXR signal can be broken down into many small segments, within which the system is approximated to behave like an LTI system, such that the system behavior can be modeled using the LDE formulation whose coefficients can be evaluated using the *least squares* (LS) method or via using neural-network modeling [29–36]. Doing so and solving for the corresponding system coefficients for each and every segment via a neural-network (see Fig. 1), the general behavior of the system coefficients can be observed over many segments, and the variance and volatility of the system coefficients can be analyzed. To do this analysis, we first start with the expression of the overall FXR signal ψ

$$\psi(n) = \lambda\left[\sum_{k=1}^{M} \Omega(n,k)\psi(k)\right] \tag{3}$$

where $\lambda[.]$ is a nonlinear operator and Ω is the time varying set of system coefficients that relates the final value n of the FXR signal to its previous values $k=\{1,2,\ldots,M\}$, $M<N$.

To perform an LTI system analysis, once again, we break down the total FXR signal into many small segments

$$\boldsymbol{\psi} = \psi(n;\xi) = [\psi(n;1), \psi(n;2), \ldots, \psi(n;P)](a\ total\ of\ P\ data\ segments) \tag{4}$$

For each data segment, we denote the local signal as $y(n)$, and perform the LTI system approximation using the LDE formulation with M coefficients

$$\psi(n,\xi) = y(n) = \sum_{k=1}^{M} a_k y(n-k) \tag{5}$$

Finally, we solve for the system coefficients (local) at a given segment with length N (which means N data samples), for the corresponding signal $\psi(n,\xi) = y(n)$, using a neural network which solves the following matrix-equation

$$\begin{bmatrix} y(n-N+1) \\ y(n-N+2) \\ \vdots \\ y(n-1) \\ y(n) \end{bmatrix} = \begin{bmatrix} y(n-N) & y(n-N-1) & \ldots & y(n-N-M+1) \\ y(n-N+1) & y(n-N) & \ldots & y(n-N-M+2) \\ \vdots & \vdots & \vdots & \vdots \\ y(n-1) & y(n-2) & \ldots & y(n-M+1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_M \end{bmatrix} \tag{6}$$

Once the coefficients are evaluated, the upcoming samples $[y(n+1), y(n+2), \ldots]$ can be estimated via Eq. 5, though with a slowly-decreasing accuracy due to the accumulation of error through each added sample. To minimize any possible error in evaluating the coefficients, we have taken N = M = 24 in this study (square matrix form), however the model can also be used for arbitrary values of N and M (N ≥ M).

Repeating the same analysis for every segment $\xi = \{1, 2, \ldots, P\}$, we obtain P values for each coefficient, which altogether constitute a coefficient matrix **A** with entries $a_{k\xi}$

$$\boldsymbol{A} = \begin{bmatrix} a_{11}, a_{12}, \ldots, a_{1P} \\ a_{21}, a_{22}, \ldots, a_{2P} \\ . \\ a_{M1}, a_{M2}, \ldots, a_{MP} \end{bmatrix}$$

Examining the segment-by-segment variation of each coefficient allows us not only to observe the local system behavior within a given time-interval, but also the inter-segmental variance and volatility of the coefficients through continuous LTI system approximation based modeling.

In the context of pure (deterministic) LDE analysis, the time variation of the coefficients will be investigated sample-by-sample, that is, the segments will be separated by a single sample, so that each new segment will be formed by a unit sample shift. Hence, each new segment differs from the adjacent segment only by a single sample. This way, the coefficients will be adaptively updated for each new sample of the FXR signal at hand. Correspondingly, we will be able to observe the sample-by-sample variation of each coefficient for a stricter volatility analysis of the FXR signal under investigation. Hence, the segments are expressed as

$$\psi(n;1) = \left[s_1, s_1, \dots, s_N\right], \psi(n;2) = \left[s_2, s_3, \dots, s_{N+1}\right], \dots, \psi(n;P) = \left[s_P, s_{P+1}, \dots, s_{N+P-1}\right]$$

3 Results

3.1 Stochastic difference-equation (AR time-series) based investigation for the EUR/USD exchange-rate

Figure 2 illustrates an AR time-series modeling based (as described in Sect. 2.1) estimation of the upcoming 24 data samples, using an apriori set of 120 data samples for the EUR/USD exchange rate, via 20 coefficients. The analyzed apriori (training) data corresponds to the period between March 2, 2017, and March 6, 2017. The data for March 9, 2017 (following weekend) is first forecasted, and then compared with the actual data for March 9, 2017 to quantify the accuracy of the estimation. The resulting estimation accuracy seems to be reasonable. The correlation coefficient between the forecasted signal and the actual signal is computed as 0.35, which indicates the accuracy of the estimation.

However, a second analysis over a later time-interval (June 2017) shows that the forecasting accuracy is not always the same. Figure 3 shows this occurence. Using 120 apriori data samples, once again the subsequent 24 data samples are estimated using AR time-series with 20 coefficients. This time the forecasted signal is quite different from the actual signal, which was naturally already known but assumed to be unknown and reserved for comparison. The correlation coefficient between the forecasted signal and the actual signal is computed as −0.16, which signifies the weakness of the accuracy. Hence, a *Monte-Carlo* analysis needs to be performed for a reliable quantification of the estimation accuracy under AR time-series modeling with various apriori data set-lengths and different number of coefficients.

Therefore, we have performed 400 consecutive estimations over the EUR/USD exchange rate signal to forecast the subsequent 24 h at each estimation [samples (1 h = 1 data sample)] using various training data-set lengths. Figure 4 shows the distribution of the correlation coefficient ρ between the actual data and the forecasted data for various number of training data samples {N = [120, 240, 480, 960]} under the use of 20 coefficients (M = 20). A near-perfect forecasting accuracy would mean that the distribution is skewed towards ρ = 1, which is not observed here in none of the cases. The distribution resembles more of a uniform-distribution rather than a Gaussian one, which would at least give some information about the

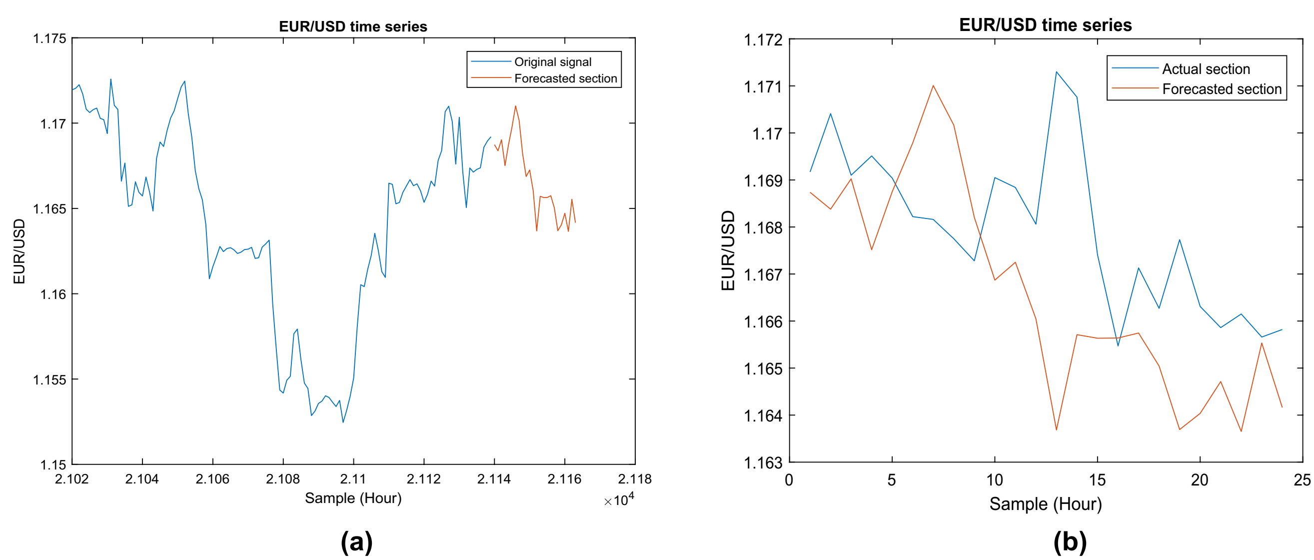


Fig. 2 **a** Forecasting the upcoming 24 data samples of the EUR/USD exchange rate using the apriori 120 data samples. **b** Comparison of the forecasted data [24 samples (hours)] with the actual data. There is reasonable forecasting accuracy with a correlation coefficient of 0.35

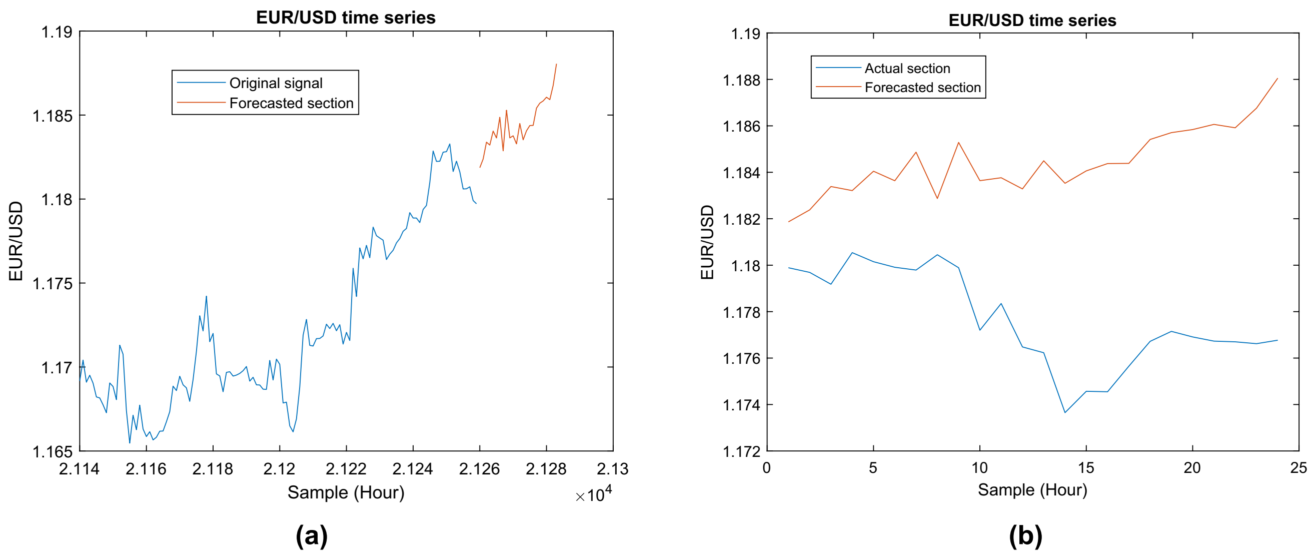


Fig. 3 **a** Forecasting the upcoming 24 data samples of the EUR/USD exchange rate using the apriori 120 data samples. **b** Comparison of the forecasted data [24 samples (hours)] with the actual data. The forecasting accuracy is weak, with a correlation coefficient of – 0.16

correlation coefficient between the actual and the forecasted data samples. From this result, it can be inferred that the estimation of future values using the AR time-series model is not precise in its most general formulation. Based on Fig. 4, the accuracy of estimation does not seem to increase by increasing the number of training data samples. It should be noted that the use of lower number of coefficients such as M = {15, 10, 5} does not seem to increase or decrease the accuracy of the estimation either, which is again indicated by the distribution of the correlation coefficients. As those cases were not noteworthy, they were not illustrated here.

Similarly, Fig. 5 shows the distribution of the correlation coefficient ρ between the actual data and the forecasted data in terms of the number of trials, for a training dataset of N = [120, 240, 480, 960] samples, and 50 coefficients. Despite the increase in the number of coefficients, the distribution is still relatively uniform with no apparent concentration near – 1 or 1. Also, there is no noteworthy difference from the distributions that were obtained using 20 coefficients. From this observation, we conclude that using an AR time-series (stochastic difference-equation [37–42]) model does not lead to an accurate estimation of future data samples, nor it does provide useful statistics that would help increasing the accuracy of the estimation. The occasional accurate estimations of future data samples are merely by chance and are not an indication of significant findings. We believe this has something to do with the high volatility of the exchange rate market, rendering forecasting almost useless. The volatility of the exchange rate market will be investigated in detail in the following section.

3.2 Deterministic difference-equation based investigation for the EUR/USD exchange-rate

Based on the formulation described in 2.2, the LTI-system coefficients of the last 960 samples of the EUR/USD exchange rate are adaptively computed via neural-network based solution. The variations of the 4th, 8th, 12th, and 16th coefficients are shown in Fig. 6 (also given in Table 1). One can notice the high volatility of the coefficients. Although, often more than 95% of the coefficient values are within the interval [– 20, 20], occasional jumps are critically high. This would mean that even one would make a reliable estimation over a certain time interval, these occasional but persistent jumps would make any estimation irrelevant as their value is much higher than the usual coefficient values, which would strongly subject the overall dynamics of the FXR signal/system to their own stochastic/chaotic behavior, rendering direct-estimation to be of minor importance compared to analysis of jump-behavior. The dynamics behind these jumps are not investigated in this study, however, the attained results show that if one has the aim of long-term exchange-rate prediction, one should focus more on determining the nature of these jumps rather than a direct forecasting approach over a section of the investigated exchange-rate signal.

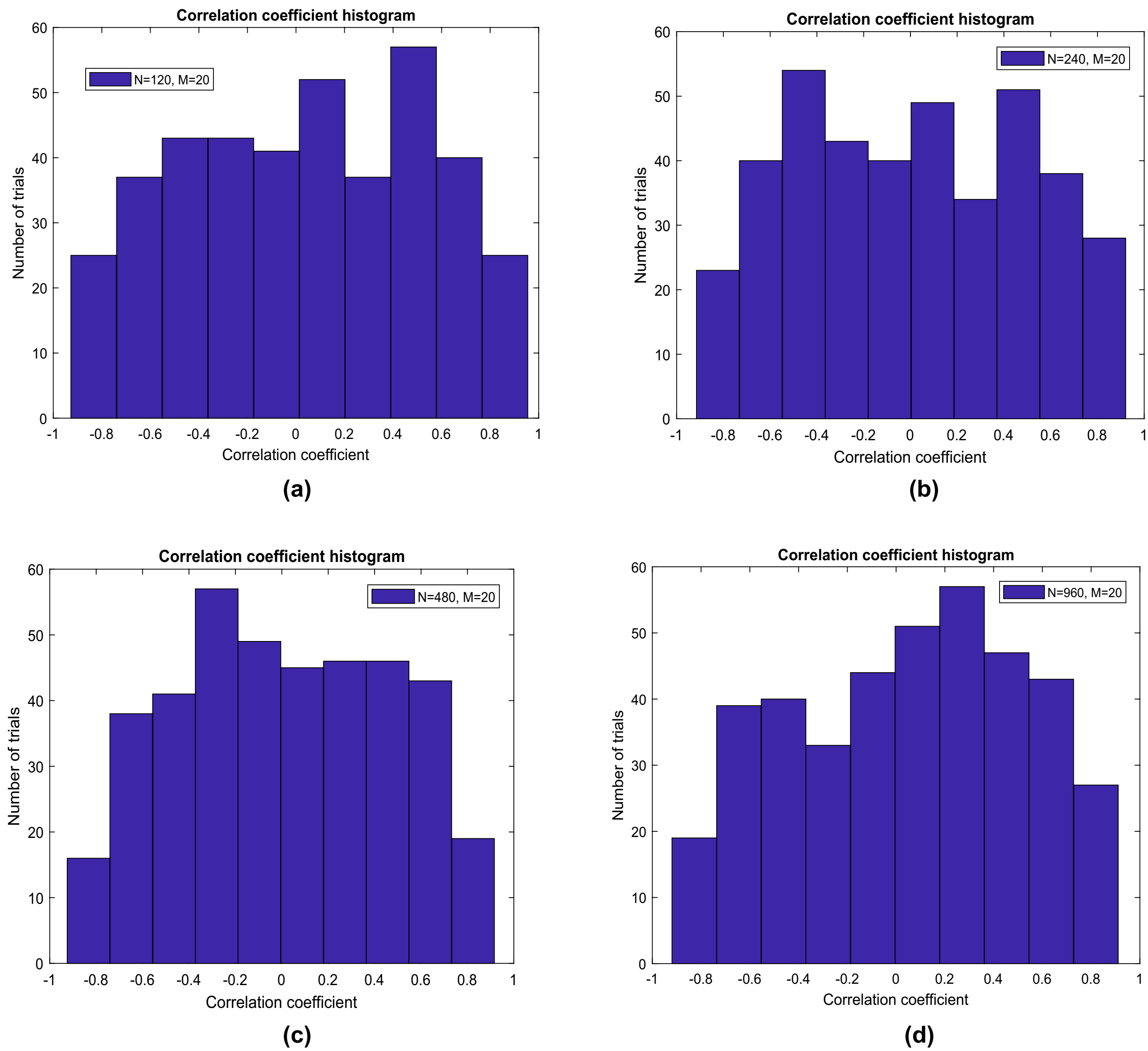


Fig. 4 Distribution of the correlation coefficient between the actual data and the estimated data for **a** N = 120, M = 20, **b** N = 240, M = 20, **c** N = 480, M = 20, **d** N = 960, M = 20. None of the distributions are skewed towards −1 or 1, but remain relatively uniform over the whole interval, which indicates no reliable estimation accuracy

3.3 Stochastic difference-equation (AR time-series) based investigation for the AUD/CAD exchange-rate

The previously described AR time-series analysis is repeated, this time for the AUD/CAD FXR signal. Figure 7 illustrates an AR time-series modeling based estimation, which again deals with the estimation of the upcoming 24 data samples, using an apriori set of 120 data samples for the AUD/CAD exchange rate, via 20 coefficients. The analyzed apriori data corresponds to the period between March 2, 2017, and March 6, 2017 as in the case for EUR/USD. The data for March 9, 2017 is forecasted and then compared with the actual data to compute the accuracy of forecasting. The resulting estimation accuracy is poor. The correlation coefficient between the forecasted signal and the actual signal is computed as 0.08, which is quite low.

As in the case of EUR/USD exchange-rate, further analysis of the AUD/CAD exchange-rate at a later date indicates a different forecasting accuracy as shown in Fig. 8. Once again, using 120 apriori data samples, the subsequent 24 data samples are estimated using AR time-series modeling with 20 coefficients at some later time-interval (June 2017). This time the forecasted signal is quite correlated

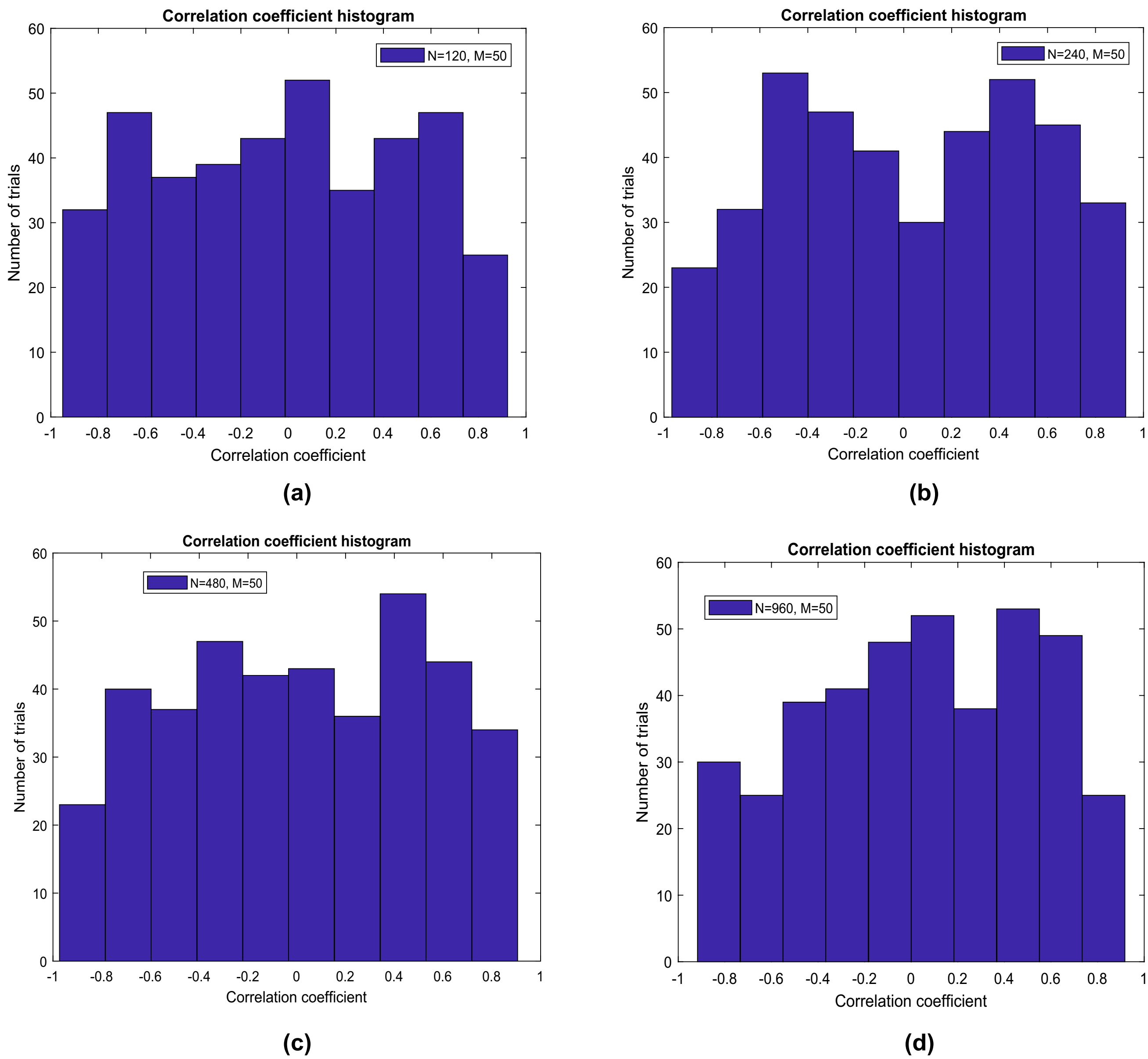


Fig. 5 Distribution of the correlation coefficient between the actual data and the estimated data for **a** N = 120, M = 50, **b** N = 240, M = 50, **c** N = 480, M = 50, **d** N = 960, M = 50. None of the distributions are skewed towards −1 or 1, but remain relatively uniform over the whole interval, which indicates no reliable estimation accuracy

with the actual signal. The correlation coefficient between the forecasted signal and the actual signal is computed as 0.83, which indicates good accuracy. Due to such conflicting results, once again, a Monte-Carlo analysis is performed to quantify the estimation accuracy under AR time-series modeling with various apriori data-set lengths and different number of coefficients.

As with the EUR/USD exchange-rate, 400 consecutive estimations over the AUD/CAD exchange-rate signal have been performed to forecast the upcoming 24 h at each estimation using various training data-set lengths. Figure 9 illustrates the distribution of the correlation coefficient ρ between the actual data and the forecasted data for various sizes of training data samples {N = [120, 240, 480, 960]} under the use of 20 coefficients (M = 20). Once again, the distribution resembles more of a uniform-distribution rather than a Gaussian one, and the distribution is not skewed towards $\rho = 1$. The accuracy of estimation does not increase via increasing the size of training data samples. The use of lower number of coefficients such as M = {15, 10, 5} does not increase/decrease the accuracy of the estimation.

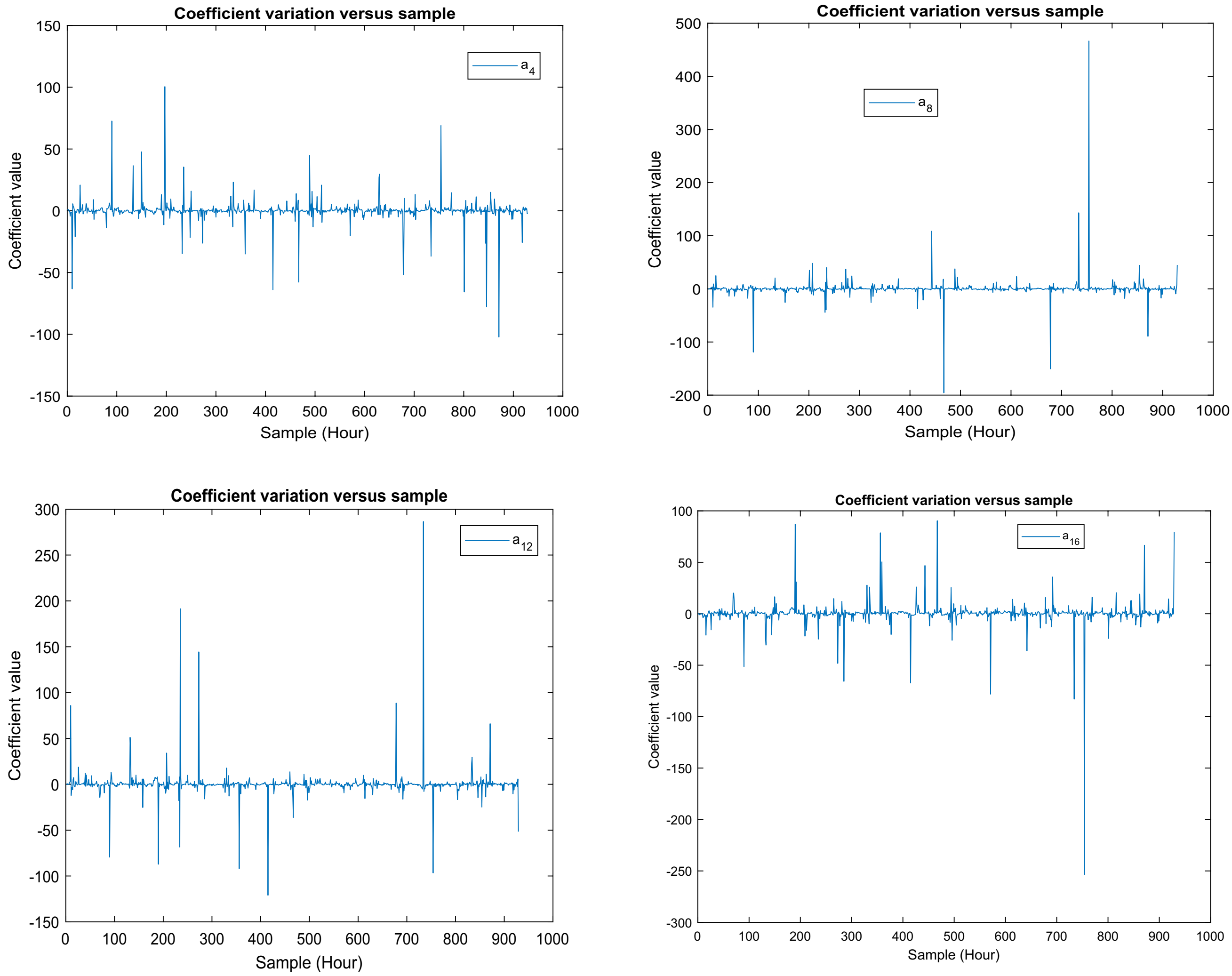


Fig. 6 Hourly variation of the 4th, 8th, 12th, and 16th coefficient for the EUR/USD exchange-rate signal

Table 1 Probability distributions of the 4th, 8th, 12th, and 16th coefficients

K	$P(a_4 < K)$	$P(a_8 < K)$	$P(a_{12} < K)$	$P(a_{16} < K)$
1	0.5748	0.5479	0.5716	0.5038
3	0.8428	0.8052	0.8105	0.8149
5	0.9042	0.8784	0.8751	0.8773
10	0.9505	0.9429	0.9494	0.9386
30	0.9806	0.9806	0.9817	0.9806
50	0.9892	0.9925	0.9849	0.9871
100	0.9978	0.9935	0.9957	0.9989

Similarly, Fig. 10 shows the distribution of the correlation coefficient ρ between the actual data and the forecasted data in terms of the number of trials, for a training dataset of N = [120, 240, 480, 960] samples, and 50 coefficients. Although the number of coefficients has greatly increased, the distribution remains mostly uniform with no sign of concentration near – 1 or 1. Furthermore, there is no significant difference from the distributions that were obtained using 20 coefficients (or less). This observation leads us to infer that the increase/decrease of the number of coefficients does not significantly affect the estimation accuracy. The estimation accuracy remains weak regardless of the number of coefficients and the training data size.

3.4 Deterministic difference-equation based investigation for the AUD/CAD exchange-rate

As in Sect. 3.2 (for the case of EUR/USD), the variations of the 4th, 8th, 12th, and 16th coefficients for the deterministic difference-equation modeling of the AUD/CAD exchange-rate, are given in Fig. 11 (also in Table 2) using the neural-network based solution. Once again, the high volatility of the

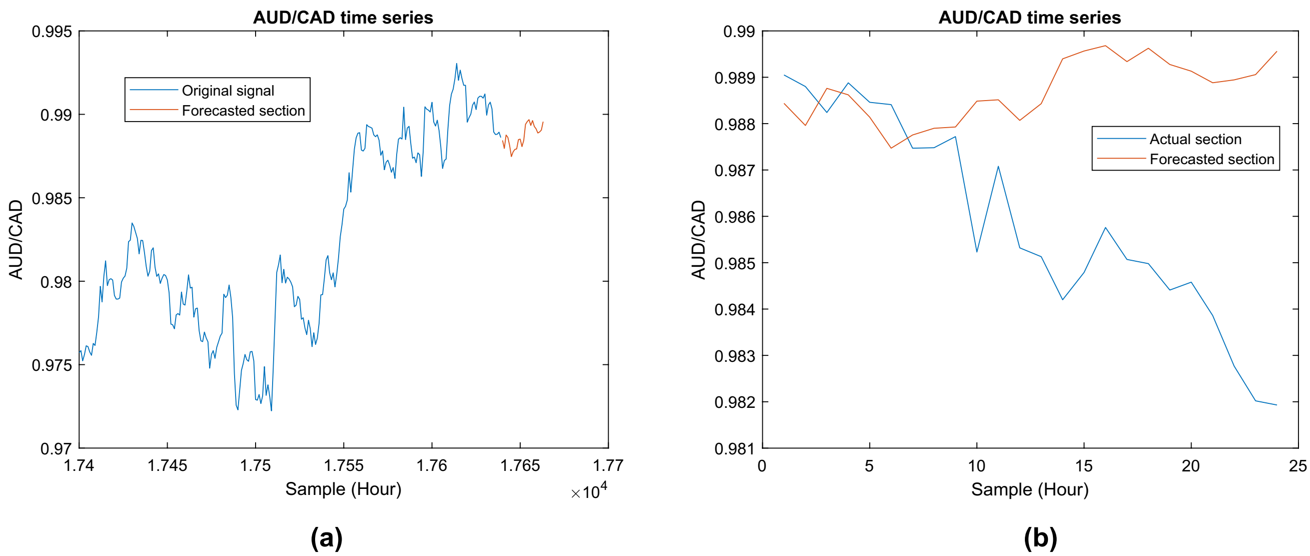


Fig. 7 **a** Forecasting the upcoming 24 data samples of the AUD/CAD exchange rate using the apriori 120 data samples. **b** Comparison of the forecasted data [24 samples (hours)] with the actual data. There is negligible forecasting accuracy with a correlation coefficient of 0.08

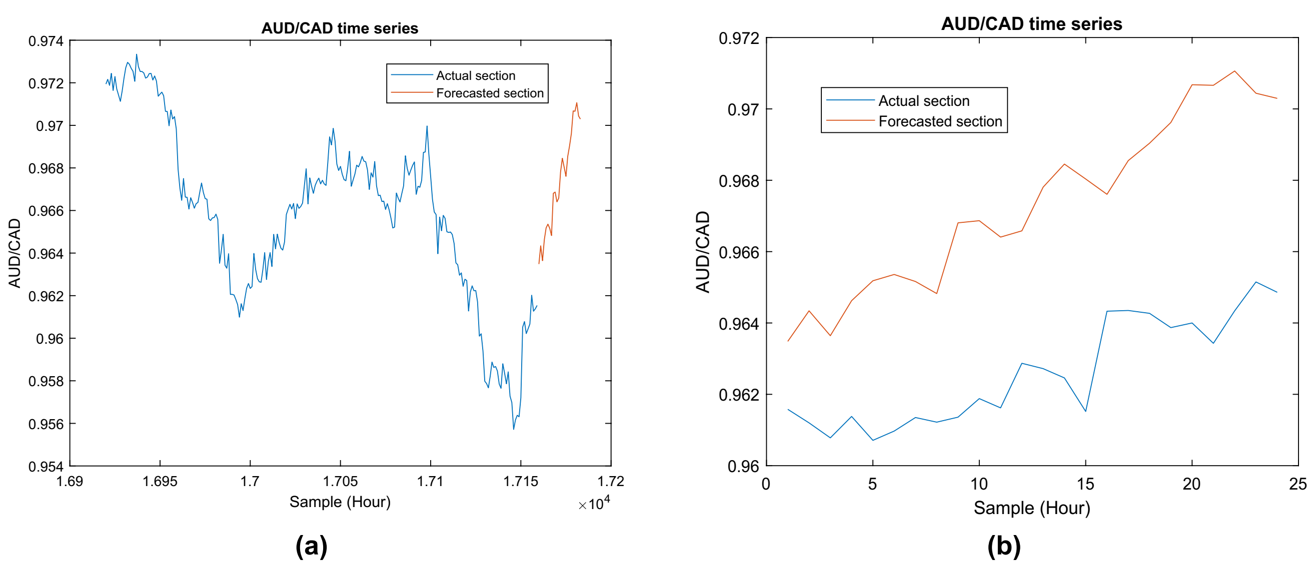


Fig. 8 **a** Forecasting the upcoming 24 data samples of the AUD/CAD exchange rate using the apriori 120 data samples. **b** Comparison of the forecasted data [24 samples (hours)] with the actual data. The forecasting accuracy is sharp, with a correlation coefficient of 0.83

coefficients is evident. Even though around 96% of the coefficient values are within the interval [– 20, 20], occasional jumps have much higher values, which prevents a reliable estimation over a certain time interval due to these persistent jumps having much higher values than usual.

4 Discussion

The uncorrelatedness between the predicted and the actual sample-sets, is understandable through the inspection of the deterministic linear difference-equation coefficients over different segments of FXR signals where the LTI-system approximation is made. The coefficient values are highly volatile and vary greatly over different segments. Most coefficients take values within the range [– 10, 10]. However, the coefficients also

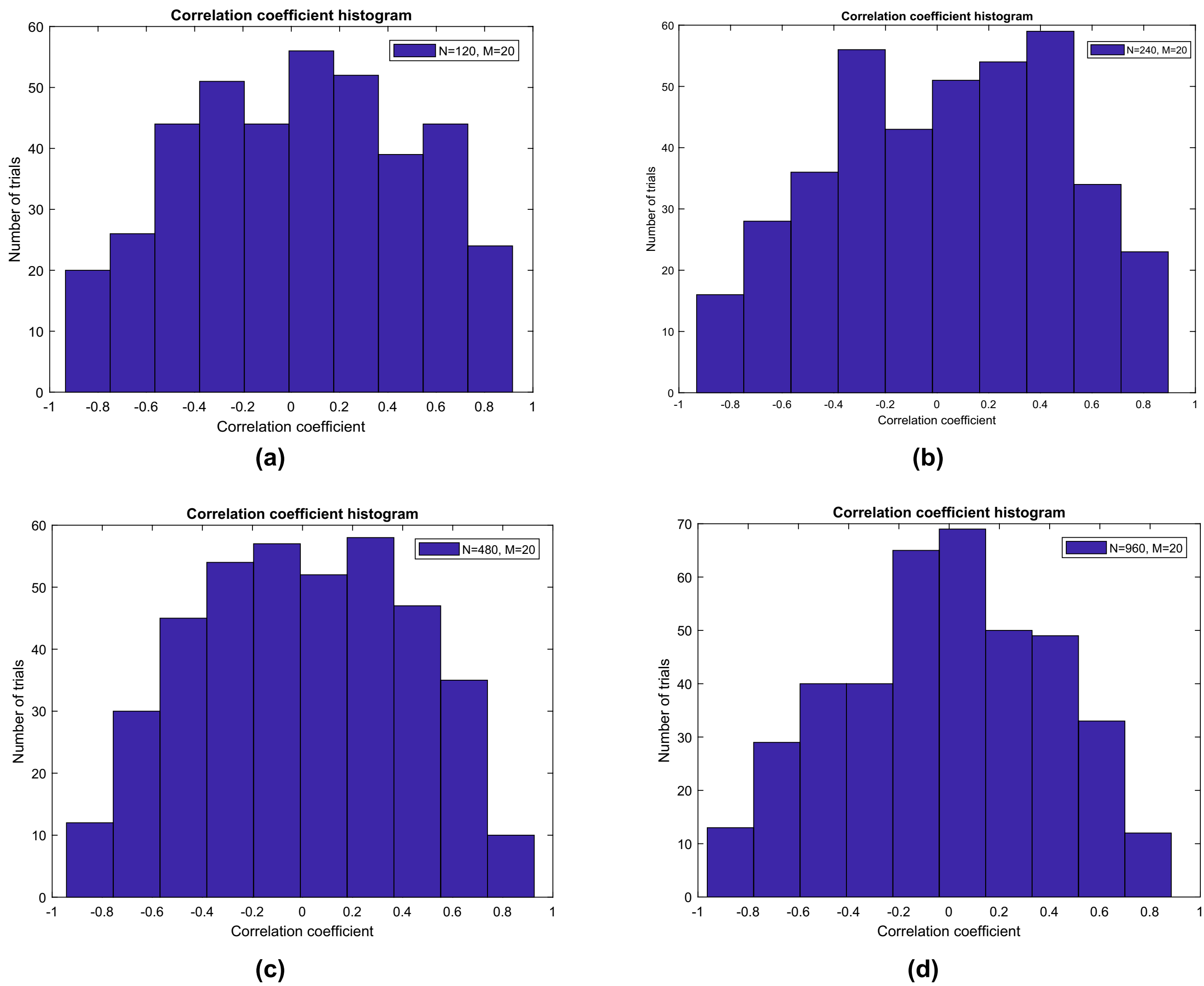


Fig. 9 Distribution of the correlation coefficient between the actual data and the estimated data (AUD/CAD) for **a** N = 120, M = 20, **b** N = 240, M = 20, **c** N = 480, M = 20, **d** N = 960, M = 20. None of the distributions are skewed towards − 1 or 1, but remain relatively uniform over the whole interval, which indicates no reliable estimation accuracy

frequently take much greater values via sudden jumps. The occurrence of these sudden jumps, makes a prediction analysis through difference-equation (DE) modeling irrelevant, as any accurate difference-equation-based estimation over a certain interval under the usual coefficient values, is greatly overshadowed by the upcoming sudden jumps, and the associated large values that the coefficients will take. Hence, it appears that an accurate prediction analysis in the FXR market is greatly dependent on the accurate estimation of these jumps and their associated dynamics, which is also supported by [28]. These sudden jumps in the coefficient values within various segments of the FXR signal at hand, do not appear to be random, but rather chaotic, as also indicated by [13, 16]. If all coefficients would make jumps around the same time-sample, a DE modeling-based prediction would be more useful within the time interval between two subsequent coefficient jumps, which necessitates the knowledge of the usual period (or the distribution of periods) between two consecutive coefficient jumps, agreeing with the observation in [13]. However, as also indicated by the observations in [14], different coefficients can make jumps over different time intervals/samples, and this suggests that a precise modeling of the jump dynamics is more useful than a DE or a time-series based modeling for prediction. Therefore, confirming the statements and findings of [28, 31], the dynamics of the coefficient-jumps mostly determine the overall dynamics of FXR signals, which are associated with certain

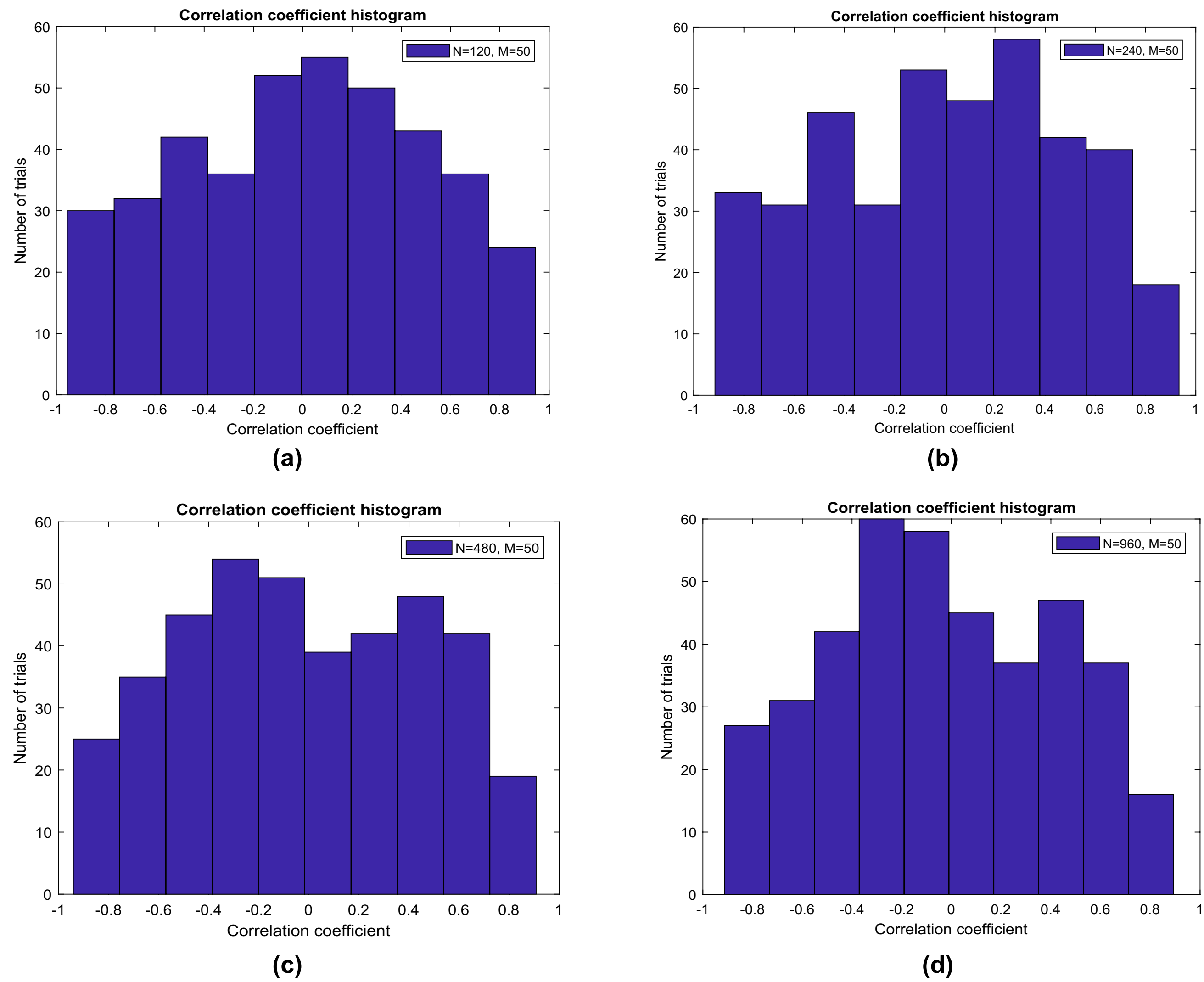


Fig. 10 Distribution of the correlation coefficient between the actual data and the estimated data for **a** N = 120, M = 50, **b** N = 240, M = 50, **c** N = 480, M = 50, **d** N = 960, M = 50. None of the distributions are skewed towards − 1 or 1, but remain relatively uniform over the whole interval, which indicates no reliable estimation accuracy

human-based social dynamics [43–45]. Hence, this study confirms the propositions and conclusions of [28, 31], and affirms the inherently chaotic and volatile nature of FXR signals in agreement with [12–16, 42].

5 Conclusion

The variation and volatility of the linear difference-equation coefficients that model a foreign exchange-rate (FXR) signal over certain time intervals where an LTI system approximation could be made, have been investigated. Initially, autoregressive (AR) time series modeling approach is used to make predictions for future values of FXR signals, over time windows that consist of 120, 240, 480, and 960 samples respectively. Through a gradual slide of the time-windows, with a total of 400 slides, and each slide corresponding to 24 samples, a Monte-Carlo analysis is made. It is shown that an accurate prediction about the future values cannot be reliably done as the distribution function of the correlation coefficient (based on 400 trials) of the predicted data and the actual data turned to be centered around 0, which indicates that the predicted sample-set have little to no similarity with the actual sample-set. Therefore, the use of a linear difference-equation with constant coefficients (as in the case of AR time series) over various segments of FXR signals turned out to be of little to no use for the estimation of future values, indicating that the coefficients should be adaptively determined for the estimation of each upcoming sample within a given segment of the signal, to increase the prediction accuracy. In this study, the degree of variation of the deterministic linear

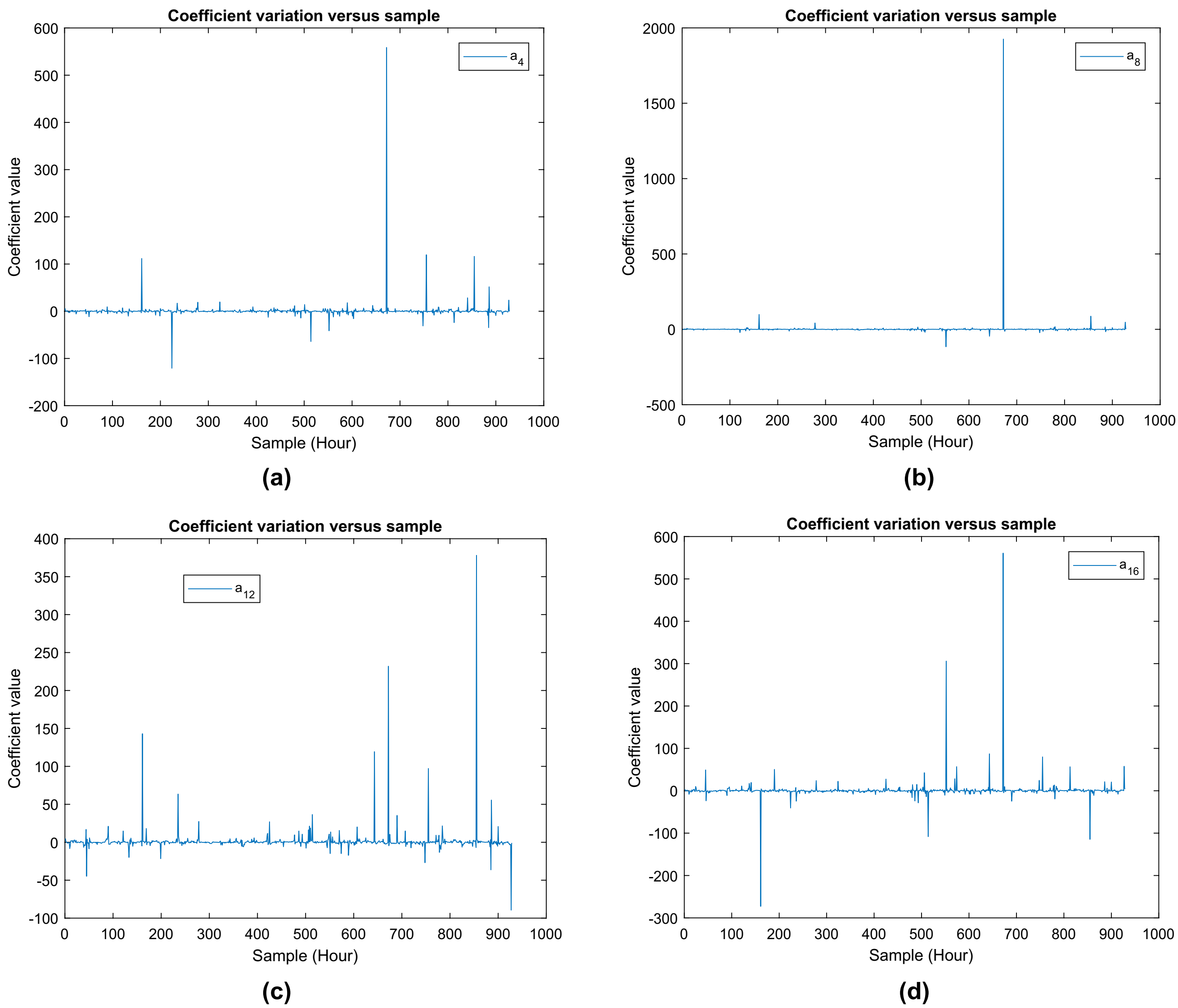


Fig. 11 Hourly variation of the 4th, 8th, 12th, and 16th coefficient for the AUD/CAD exchange rate signal

Table 2 Probability distribution of the 4th, 8th, 12th, and 16th coefficient for the AUD/CAD exchange rate

K	$P(a_4 < K)$	$P(a_8 < K)$	$P(a_{12} < K)$	$P(a_{16} < K)$
1	0.5996	0.6243	0.5834	0.5059
3	0.8654	0.8784	0.8601	0.8342
5	0.9214	0.9257	0.9150	0.8988
10	0.9688	0.9709	0.9580	0.9548
30	0.9892	0.9925	0.9871	0.9849
50	0.9925	0.9946	0.9914	0.9882
100	0.9946	0.9957	0.9957	0.9946
300	0.9989	0.9978	0.9978	0.9978
600	1	0.9989	1	1

difference-equation coefficients have also been demonstrated, for two foreign exchange-rate (EUR/USD, AUD/CAD) signals, via segmentation of the overall exchange-rate signals into smaller sample-sets, enabling LTI system representation. After solving for the local (LTI system) coefficients of the signal via neural-network-modeling, and the analysis of all segments (sample-sets), the linear difference-equation coefficients are observed to be far from being constant or quasi-constant over different segments. Specifically, the coefficients are mostly observed to attain values within the range [– 10, 10], though, can occasionally but frequently assume values that are way greater than 100 via sudden jumps. This shows that the coefficients are highly volatile, making prediction/estimation difficult via difference-equation modeling, as the seemingly chaotic sudden jumps appear to greatly influence the dynamics of the underlying FXR system/market. Our conclusion suggests that the prediction of future values of an FXR signal is quite difficult due to this volatility. Nevertheless, knowing the distribution functions and the uppermost/lowermost values of the linear difference-equation coefficients could help risk-management while trading in the FX market.

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Code availability The code is available in the article.

Declaration

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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