Phys. Lett. B 849 (2024) 138424

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

Letter

A model for time-evolution of coupling constants

Taekoon Lee

Department of Physics, Kunsan National University, Gunsan 54150, Korea

ARTICLE INFO

Editor: G.F. Giudice

ABSTRACT

A general model is proposed for time-varying coupling constants in field theory, assuming the ultraviolet cutoff is a varying entity in the expanding universe. It is assumed that the cutoff depends on the scale factor of the universe and all bare couplings remain constant. This leads to varying renormalized coupling constants that evolve in proportion to the Hubble parameter. The evolution of the standard model constants is discussed.

The fundamental laws of physics are generally considered universal, independent of time and location. While this universality is taken for granted locally, it is an intriguing question whether it extends to the entire history of the universe. Noting the similarity in the large ratios, of order 10^{39}, of the electric to gravitational forces between an electron and a proton and the age of the universe in atomic time units, Dirac proposed that such a large number has to do with the age of the universe, with the gravitational constant being inversely proportional to it [1]. While this turned out to be incompatible with various evidences, including the geological ones [2], the question on the constancy of physical constants over a cosmological time scale has been of great interest ever since [3,4].

The implication of time dependence of the physical laws would unquestionably be revolutionary. Above all, varying constants violate the equivalence principle of general relativity that the physical laws be same to all free falling observers, which would require a modification of the general relativity, and perhaps a fundamental change in our understanding of the spacetime.

Being accurately measurable the fine structure constant α has attracted a special attention in the study of time-variation of physical constants, and a broad class of theoretical models on varying α is based on scalar fields, which include the dilaton from the string theory or a Kaluza-Klein mode of higher-dimensional spacetime models. In these models the scalar field couples to the electromagnetic fields non-minimally, and the evolution of the scalar field renders the fine structure constant to vary in time and space. In other models based on grand unified theories (GUTs) a varying fine structure constant would imply varying gauge couplings of the strong, weak interactions as well, resulting in a varying nucleon mass [5,6].

In this note we propose a mechanism for time-varying coupling constants in field theories in general. It may appear that a field theory such as the standard model would not allow varying coupling constants within its framework. There is however a venue: the ultraviolet (UV) cutoff. In a renormalizable theory of elementary particles the cutoff is usually introduced to regularize the divergence of loop amplitudes. The cutoff could be an artifact of renormalization process, and in asymptotically free theories like the quantum chromodynamics (QCD) it needs not have a physical meaning and can be put to an infinite limit. On the other hand in a theory like quantum electrodynamics (QED), which is thought to be a trivial theory in the infinite cutoff limit, the cutoff should be finite for the theory to have non-vanishing interactions. In this case the cutoff would be physical and the theory must be considered as a low energy effective theory. In this note we consider a field theory as an effective theory and assume the UV cutoff is finite and physical. We further suppose the cutoff is also time-varying. Since the renormalizable couplings are related to the bare couplings at the cutoff scale via the renormalization group equations (RGEs), they will vary as the cutoff varies, provided the bare couplings remain constant.

As an illustration let us consider the QCD. Because the QCD is asymptotic free and UV safe, the cutoff may be rendered to infinity. In the chiral limit, where quarks are massless, the nucleon mass m_n can be written in the form

$$m_n = \Lambda f(g_\Lambda) \tag{1}$$

where Λ, g_Λ are the UV cutoff and the bare coupling, respectively, and f is a function given by the beta function of the coupling. In renormalization theory, m_n is invariant under the variation of the cutoff because the bare coupling varies as well according to the RGE so that m_n remains constant. We may now assume, however, the QCD is an effective theory built on a lattice, like the lattice QCD, but the cutoff, the inverse of lattice spacing, is physical, and time-dependent, while the bare coupling remains at a fixed but small value so that the theory remains near

E-mail address: tlee@kunsan.ac.kr.

https://doi.org/10.1016/j.physletb.2023.138424
Received 29 October 2023; Received in revised form 22 December 2023; Accepted 22 December 2023

Available online 28 December 2023
0370-2693/© 2023 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).

the continuum limit. The nucleon mass will then vary in proportion to the cutoff as given in Eq. (1) — a varying nucleon mass.

This idea can be easily implemented on a general renormalizable theory. Let us consider the RGEs for a set of N dimensionless couplings $\alpha_i(\mu)$, where $i = 1, 2, \cdots, N$:

$$\mu \frac{d\alpha_i(\mu)}{d\mu} = \beta_i(\alpha_1(\mu), \cdots, \alpha_N(\mu)), \quad (2)$$

where the couplings and the beta functions are from a mass-independent renormalization scheme like the MS scheme. We shall now assume that the cutoff varies under the expansion of the universe, while the bare couplings $\alpha_i^{\mathrm{B}} \equiv \alpha_i(\Lambda)$ remain constant. To see the variation of $\alpha_i(\mu)$ under this boundary condition we write the solution of Eq. (2) in the form:

$$\alpha_i(\mu) = F_i(\alpha_1^{\mathrm{B}}, \cdots, \alpha_N^{\mathrm{B}}, \log(\Lambda/\mu)),$$

from which we get

$$\begin{aligned} \delta\alpha_i(\mu) &= \frac{\partial F_i}{\partial \log\Lambda}\frac{\delta\Lambda}{\Lambda} \\ &= -\frac{\partial F_i}{\partial \log\mu}\frac{\delta\Lambda}{\Lambda} \\ &= -\beta_i(\alpha_1(\mu), \cdots, \alpha_N(\mu))\frac{\delta\Lambda}{\Lambda}. \end{aligned} \quad (3)$$

Note that the variation of the couplings at the scale μ is local in that it depends only on the beta function values at the same scale, oblivious of the evolution history of the couplings. In deriving Eq. (3) we assumed there was no threshold between the cutoff and μ. In case there is one Eq. (3) can be used to compute the variation of the coupling at just above the matching scale, then using the matching function obtain the coupling just below it, and run the coupling down to obtain the variation at μ.

What causes the cutoff to vary? We cannot answer it but there have been speculations that the spacetime may be discrete and lattice-like at a short distance. If this is the case then the lattice spacing would be the cutoff, and it may not be inconceivable that the lattice varies as well, as the universe expands, causing the cutoff to vary. Whatever the cause may be we assume that the cutoff varies in the expanding universe, and the variation depends on the scale factor of the Robertson-Walker metric. As a simple ansatz we shall assume a power-law variation,

$$\Lambda \propto a(t)^{-\kappa}, \quad (4)$$

where κ is a constant and $a(t)$ is the scale factor. Since κ determines the strength of the time-dependence of the cutoff, we may think of it as a measure of the rigidity of the field theory in the expanding universe.

With the ansatz we have

$$\dot{\Lambda}/\Lambda = -\kappa H, \quad (5)$$

where H is the Hubble parameter. Then with (3) we get

$$\frac{d\alpha_i(\mu)}{dt} = \kappa\beta_i(\alpha_1(\mu), \cdots, \alpha_N(\mu))H, \quad (6)$$

which shows the sign of variation of a coupling constant is dependent on its beta function. Thus the fine structure constant and the strong coupling constant of QCD evolve in the opposite direction as their beta functions have opposite signs. With Eq. (6) the variation of the coupling from the epoch of red shift z to *now* is given by

$$\Delta\alpha_i \equiv \alpha_i^z(\mu) - \alpha_i^0(\mu) = -\kappa\beta_i(\alpha_1^0(\mu), \cdots, \alpha_N^0(\mu))\log(1+z) + \mathcal{O}(\kappa^2), \quad (7)$$

where α_i^0 are the couplings at present. So the couplings vary logarithmically in time as κ is expected to be very small, and for a large z, $1+z$ is proportional to $t^{-2/3}$ and $t^{-1/2}$ for the matter dominated and radiation dominated epochs, respectively.

In the following we investigate the implication of the result on the standard model couplings, using the leading-order beta functions. An immediate consequence of our scenario is that the renormalized couplings have time-variations that are all related, as the variations arise from a single cutoff.

The fine structure constant: The standard model couplings α_i ($i = 1, 2, 3$) run at one-loop order by

$$\mu\frac{d\alpha_i}{d\mu} = b_i\alpha_i^2, \quad (8)$$

where $b_i = (41/6, -19/6, -7)/2\pi$ for $\mu > M_Z$, and $\alpha_i = g_i^2/4\pi$, g_i the gauge couplings of the $U(1)_Y \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(3)_C$. At the leading order the threshold effects in the running coupling below the electroweak symmetry breaking scale can be ignored, and the variation of the fine structure constant α can be written in terms of those of the electroweak couplings at $\mu = M_Z$. Then we get

$$\delta(1/\alpha) = \delta(1/\alpha_1(M_Z)) + \delta(1/\alpha_2(M_Z)) = \frac{11}{6\pi}\frac{\delta\Lambda}{\Lambda},$$

and

$$\dot{\alpha}/\alpha = \frac{11\kappa}{6\pi}\alpha H, \quad \Delta\alpha/\alpha = -\frac{11\kappa}{6\pi}\alpha\log(1+z). \quad (9)$$

The parameter κ can be constrained by astronomical observations as well as laboratory experiments. The only observation of variation of the fine structure constant that is significantly different from zero is by Webb et al. from the quasar absorption spectra [7,8], with weighted mean:

$$\Delta\alpha/\alpha = (-0.57 \pm 0.11) \times 10^{-5}, \quad \text{for} \quad 0.2 < z < 4.2,$$

which gives

$$0.66 \times 10^{-3} < \kappa < 0.88 \times 10^{-2}. \quad (10)$$

There are many other studies that do not confirm the time-variation of α [3], and it would be safe to assume that

$$|\kappa| \lesssim 10^{-2}.$$

A laboratory experiment using atomic clocks can also give a stringent limit. The experiment on the variation of the frequency ratio of Al^+ and Hg^+ single ion optical clocks yields a bound [9],

$$\dot{\alpha}/\alpha = (-1.6 \pm 2.3) \times 10^{-17}/\text{year},$$

which gives

$$\kappa = (-5.4 \pm 7.8) \times 10^{-5}, \quad (11)$$

and the latest experiment on the frequency ratio of the electro-octupole and electro-quadrupole transitions in Yb^+ yields [10]

$$\dot{\alpha}/\alpha = (1.8 \pm 2.5) \times 10^{-19}/\text{year},$$

which gives

$$\kappa = (6.1 \pm 8.5) \times 10^{-7}. \quad (12)$$

The strong coupling constant and the nucleon mass: A similar calculation for the strong coupling constant gives

$$\delta(1/\alpha_3(\mu)) = -\frac{7}{2\pi}\delta\Lambda/\Lambda, \quad \dot{\alpha}_3/\alpha_3 = -\frac{7\kappa}{2\pi}\alpha_3 H, \quad (13)$$

where μ is of the nucleon mass scale. In the chiral limit the nucleon mass m_n is proportional to Λ_{QCD} given by

$$1/\alpha_3(\mu) = \beta_0\log(\mu/\Lambda_{\mathrm{QCD}}), \quad (14)$$

where $\beta_0 = 9/2\pi$ for three light-quark flavors. Therefore,

$$\frac{\delta m_n}{m_n} = \frac{\delta\Lambda_{\mathrm{QCD}}}{\Lambda_{\mathrm{QCD}}} = \frac{7}{9}\frac{\delta\Lambda}{\Lambda}, \quad (15)$$

which shows the variation of the nucleon mass mirrors that of the cutoff and gives

$$\frac{\dot{m}_n}{m_n} = -\frac{7}{9}\kappa H\,, \quad m_n^z = m_n^0\left[1+\frac{7}{9}\kappa\log(1+z)\right], \tag{16}$$

where m_n^z is the nucleon mass at redshift z. The ratio of the nucleon mass variation to that of the fine structure constant, which has a strong model dependence, is given by

$$R = \left(\frac{\dot{m}_n}{m_n}\right)\bigg/\left(\frac{\dot{\alpha}}{\alpha}\right) = -\frac{14\pi}{33\alpha} = -183\,,$$

which compares to the SU(5) GUT model value $R = 36$ [5,6].

The lepton, quark, and Higgs masses: The running mass for these particles is given in the form

$$m(\mu) = \xi(\mu)v(\mu)\,, \tag{17}$$

where v denotes the Higgs vacuum expectation value, ξ the Yukawa coupling for the lepton and quark mass, and $\xi = \sqrt{\lambda}$ for the Higgs mass, where λ is the Higgs quartic coupling. The RG equation for m is given by

$$\mu\frac{dm(\mu)}{d\mu} = \gamma_m(\mu)m(\mu)\,, \tag{18}$$

where $\gamma_m(\mu) = \gamma_\xi(\mu) + \gamma_v(\mu)$ with

$$\mu\frac{d\xi(\mu)}{d\mu} = \xi(\mu)\gamma_\xi(\mu)\,, \quad \mu\frac{dv(\mu)}{d\mu} = v(\mu)\gamma_v(\mu)\,.$$

Similarly to the variation of the couplings in (3) the mass variation is given by

$$\frac{\delta m(\mu)}{m(\mu)} = -\gamma_m(\mu)\frac{\delta\Lambda}{\Lambda}\,, \quad \frac{\dot{m}}{m} = \kappa\gamma_m H\,. \tag{19}$$

The anomalous dimension for v can be written as

$$\gamma_v = (\gamma_{m_H^2} - \beta_\lambda/\lambda)/2\,, \tag{20}$$

where $\gamma_{m_H^2}$ is the anomalous dimension of the Higgs mass squared and β_λ is the beta function for λ.

For the electron mass m_e the anomalous dimension is given by

$$\gamma_{m_e} = -\frac{3}{16\pi^2}\left[\lambda + g_1^2 + \frac{1}{8\lambda}(g_1^4 + 2g_1^2g_2^2 + 3g_2^4 - 16Y_t^4)\right], \tag{21}$$

where Y_t is the top quark Yukawa coupling [11]. Evaluating it at $\mu = M_Z$ with $g_1 = 0.350, g_2 = 0.653, Y_t = 0.935$, and $\lambda = 0.265$ we have

$$\frac{\delta m_e}{m_e} = -0.096\frac{\delta\Lambda}{\Lambda}\,, \tag{22}$$

which is about 1/8th of the nucleon mass variation of the opposite sign.

The variation of the proton-to-electron mass ratio, $\zeta = m_p/m_e$, is given by

$$\dot{\zeta}/\zeta = -0.874\kappa H\,, \quad \Delta\zeta/\zeta = 0.874\kappa\log(1+z)\,.$$

The variation for ζ can be constrained by molecular transition lines, and Reinhold et al. observed a non-vanishing value $\Delta\zeta/\zeta = (2.4\pm0.6)\times10^{-5}$ from a weighted fit of H_2 spectral lines from two quasars at redshift $z = 3.02$ and $z = 2.59$ [12], which yields $\kappa = (2.4 \pm 0.6) \times 10^{-5}$. However, the inversion spectrum of ammonia yields a tighter constraint of $|\Delta\zeta/\zeta| < 1.8 \times 10^{-6}$ from the absorption lines of quasars at redshift $z = 0.685$, which gives $\kappa < 4 \times 10^{-6}$ [13]. The laboratory experiments probing frequency drift of atomic clocks also give stringent constraints on the differential variation: $\dot{\zeta}/\zeta = (-5.3 \pm 6.5) \times 10^{-17}$/yr [14], yielding $\kappa = (0.9 \pm 1.1) \times 10^{-6}$.

The gravitational constant G: The Planck scale $M_{\rm pl} = 1/\sqrt{G}$ may be a natural candidate for the UV cutoff. If it is the case then the gravitational constant varies as

$$\dot{G}/G = -2\dot{\Lambda}/\Lambda = 2\kappa H\,, \quad \Delta G/G = -2\kappa\log(1+z)\,, \tag{23}$$

with G varying in power law, for example, $G(t) \propto t^{\frac{4}{3}\kappa}$ in the matter dominated epoch. A constraint on the power law variation of G has been placed from helioseismology of the sun that probes its evolution under varying G [15], from which we get $|\kappa| \lesssim 0.1$. The big bang nucleosynthesis also provides a constraint on the power law variation [16], which corresponds to $|\kappa| \lesssim 0.008$.

In summary we proposed a model for time varying coupling constants in a renormalizable field theory, assuming the UV cutoff is varying in the expanding universe while the bare couplings remain fixed. This renders the renormalized couplings to vary in time per the renormalization group equations. The evolution of the couplings is proportional to the Hubble parameter and the beta functions of the couplings. The time rates of the couplings are not independent but related by the beta functions, as they arise from a single UV cutoff. Of the standard model constants the most sensitive to the cutoff evolution is the nucleon mass, which evolution mirrors the cutoff.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgements

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2019R1F1A1063011) and KNU.

References

[1] P.A.M. Dirac, Nature 139 (1937) 323.
[2] E. Teller, Phys. Rev. 73 (1948) 801.
[3] J.-P. Uzan, Living Rev. Relativ. 14 (2011) 2, arXiv:1009.5514 [astro-ph.CO].
[4] C.J.A.P. Martins, Rep. Prog. Phys. 80 (2017) 126902.
[5] X. Calmet, H. Fritzsch, Phys. Lett. B 540 (2002) 173.
[6] M. Dine, Y. Nir, G. Raz, T. Volansky, Phys. Rev. D 67 (2003) 015009.
[7] J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, J.D. Barrow, Phys. Rev. Lett. 82 (1999) 884.
[8] M.T. Murphy, J.K. Webb, V.V. Flambaum, Mon. Not. R. Astron. Soc. 345 (2003) 609, arXiv:astro-ph/0306483.
[9] T. Rosenband, D.B. Hume, P.O. Schmidt, C.W. Chou, A. Brusch, L. Lorini, W.H. Oskay, R.E. Drullinger, T.M. Fortier, J.E. Stalnaker, S.A. Diddams, W.C. Swann, N.R. Newbury, W.M. Itano, D.J. Wineland, J.C. Bergquist, Science 319 (2008) 1808, https://www.science.org/doi/pdf/10.1126/science.1154622.
[10] M. Filzinger, S. Dörscher, R. Lange, J. Klose, M. Steinel, E. Benkler, E. Peik, C. Lisdat, N. Huntemann, Phys. Rev. Lett. 130 (2023) 253001.
[11] M.-x. Luo, Y. Xiao, Phys. Rev. Lett. 90 (2003) 011601, arXiv:hep-ph/0207271.
[12] E. Reinhold, R. Buning, U. Hollenstein, A. Ivanchik, P. Petitjean, W. Ubachs, Phys. Rev. Lett. 96 (2006) 151101.
[13] M.T. Murphy, V.V. Flambaum, S. Muller, C. Henkel, Science 320 (2008) 1611, https://www.science.org/doi/pdf/10.1126/science.1156352.
[14] W.F. McGrew, X. Zhang, H. Leopardi, R.J. Fasano, D. Nicolodi, K. Beloy, J. Yao, J.A. Sherman, S.A. Schäffer, J. Savory, R.C. Brown, S. Römisch, C.W. Oates, T.E. Parker, T.M. Fortier, A.D. Ludlow, Optica 6 (2019) 448.
[15] P. Demarque, L.M. Krauss, D.B. Guenther, D. Nydam, Astrophys. J. 437 (1994) 870.
[16] F.S. Accetta, L.M. Krauss, P. Romanelli, Phys. Lett. B 248 (1990) 146.